Graviton Corrections to Maxwell's Equations

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Classical Maxwell's Equations

$\partial_\nu [\sqrt{-g}\, g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}] = J^\mu$

1. Photons ($J^\mu = 0$)
   - $E^i(t,x) \sim \varepsilon^i e^{-ik(ct-x)}$
2. Static point charge ($J^\mu = q\delta^\mu{}_0 \delta^3(x)$)
   - $A_0(t,x) = q/[4\pi r]$
3. Instantaneous dipole
   - $J^0 = -\theta(t) p^i \partial_i \delta^3(x)$, $J^i = p^i \delta(t) \delta^3(x)$
   - $F_{ij}(t,x\} = -1/2\pi\, (\partial_i p_j - \partial_j p_i)\, \theta(t)\, \delta(r^2 - t^2)$

Quantum Effective Field Eqns

- For $S[\varphi] \rightarrow \Gamma[\varphi_{cl}]$
- Exact Heisenberg Eqn $\delta S[\varphi]/\delta\varphi(x) = 0$
- Want $\varphi_{cl}(x) = \langle\Psi_L|\varphi(x)|\Psi_R\rangle$
- $\delta\Gamma[\varphi_{cl}]/\delta\varphi_{cl}(x) =$

  $$\langle\Psi_L|\delta S[\varphi_{cl}+\Delta\varphi]/\delta\varphi(x)|\Psi_R\rangle$$

  - Assuming $\langle\Psi_L|\Delta\varphi(x)|\Psi_R\rangle = 0$
  - Propagators & vertices in φ_{cl}

Different Choices for Ψ_L & Ψ_R

- In-Out ➔ $\Psi_{L,R} \sim$ free as $t \to \pm\infty$
  - Right for scattering
  - But neither real nor causal
- In-In ➔ $\Psi_L = \Psi_R \sim$ free as $t \to -\infty$
  - Both real and causal
  - Read off from in-out with Schwinger-Keldysh
- Hybrid ➔ $\Psi_L = \Psi_R$ at $t = 0$
  - Start interactions at $t = 0$
  - Perturbatively correct initial state

Quantum-Correcting Maxwell

- $\partial_\nu[\sqrt{-g}\, g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)]$
  $+ \int d^4x' [{}^\mu\Pi^\nu](x;x')\, A_\nu(x') = \mathcal{J}^\mu(x)$
  - 1PI 2-pt function: $i[{}^\mu\Pi^\nu](x;x')$
  - 1PI 1-pt function: $\mathcal{J}^\mu(x)$
- Specialization to flat space
  - $[{}^\mu\Pi^\nu](x;x') = (\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)\, \Pi(\Delta x^2)$
  - $\partial_\nu F^{\nu\mu}(x) + \int d^4x'\, \Pi(\Delta x^2)\, \partial'_\nu F^{\nu\mu}(x') = \mathcal{J}^\mu(x)$

Perturbative Solution in $\kappa^2 = 16\pi G$

- Expand everything in powers of κ^2
  - $F^{\mu\nu} = F_0^{\mu\nu} + \kappa^2 F_1^{\mu\nu} + \kappa^4 F_2^{\mu\nu} \ldots$
  - $\mathcal{J}^\mu = J_0^\mu + \kappa^2 J_1^\mu + \kappa^4 J_2^\mu + \ldots$
  - $\Pi(\Delta x^2) = 0 + \kappa^2 \Pi_1(\Delta x^2) + \kappa^4 \Pi_2(\Delta x^4) + \ldots$
- Segregate powers of κ^2
  - $\partial_\nu F_0^{\mu\nu}(x) = J_0^\mu(x)$
  - $\partial_\nu F_1^{\mu\nu}(x) = J_1^\mu(x) - \int d^4x' \, \Pi(\Delta x^2) \, \partial'_\nu F_0^{\mu\nu}(x')$

$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{EM} + \mathcal{L}_{BPHZ}$

- $\mathcal{L}_{GR} = 1/16\pi G\, R\sqrt{-g}$
- $\mathcal{L}_{EM} = -\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$

  $\delta^2 S_{EM}/\delta A_\mu(x)\delta A_\rho(x') = \partial_\kappa\{\sqrt{-g}[g^{\kappa\lambda}g^{\mu\rho} - g^{\kappa\rho}g^{\lambda\mu}]\partial_\lambda \delta^D(x-x')\}$
- $\mathcal{L}_{BPHZ} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$
- $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \quad \kappa^2 = 16\pi G$
  - $\sqrt{-g}\,[g^{\kappa\lambda}g^{\mu\rho} - g^{\kappa\rho}g^{\lambda\mu}] = \eta^{\kappa\lambda}\eta^{\mu\rho} - \eta^{\kappa\rho}\eta^{\lambda\mu}$

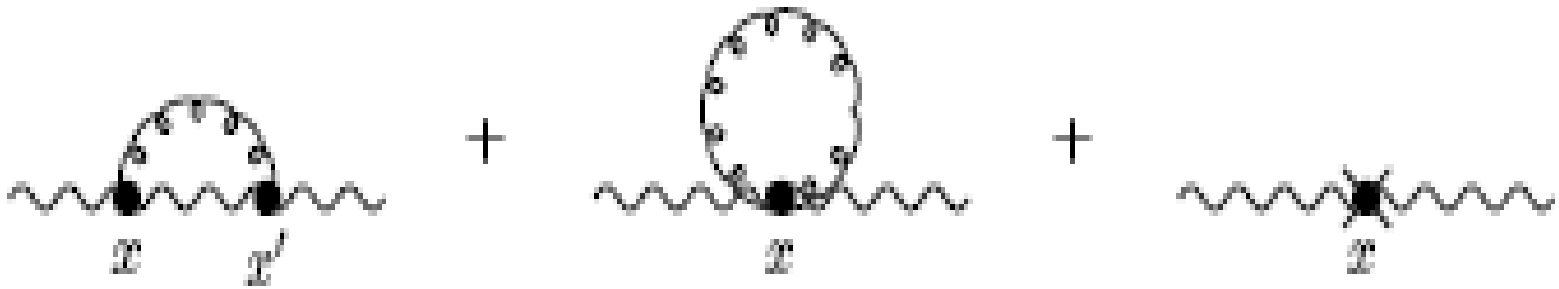
    $+ \kappa V^{\mu\rho\kappa\lambda\alpha\beta} h_{\alpha\beta} + \kappa^2 U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta} h_{\alpha\beta} h_{\gamma\delta} + O(\kappa^3)$
  - $V^{\mu\rho\kappa\lambda\alpha\beta} = \eta^{\alpha\beta}\eta^{\kappa[\lambda}\eta^{\rho]\mu} + 4\,\eta^{\alpha)[\mu}\eta^{\kappa][\rho}\eta^{\lambda](\beta}$

All Propagators from Scalar

(dimensionally regulate in D dimensions)

- $i\Delta(x;x') = \Gamma(D/2 -1)/4\pi^{D/2} (1/\Delta x^2)^{D/2 -1}$
  - $\Delta x^2 = ||x - x'||^2 - (|t - t'| - i\varepsilon)^2$
- $\mathcal{L}_{EMfix} = -½ (\partial^\mu A_\mu)^2$
  - $i[_\mu\Delta_\nu](x;x') = \eta_{\mu\nu}\, i\Delta(x;x')$
- $\mathcal{L}_{GRfix} = -½ (\partial^\nu h_{\mu\nu} - ½\partial_\mu h)^2$
  - $i[_{\mu\nu}\Delta_{\rho\sigma}](x;x')$
    $= [2\eta_{\mu(\rho}\eta_{\sigma)\nu} - 2/(D-2)\, \eta_{\mu\nu}\eta_{\rho\sigma}]\, i\Delta(x;x')$
- NB have both real and imaginary parts

No Integrals at One Loop!



$$i[{}^{\mu}\Pi^{\nu}]_{3pt} = \partial_{\kappa}\partial'_{\theta}\{i\kappa V^{\mu\rho\kappa\lambda\alpha\beta}\ i[{}_{\alpha\beta}\Delta_{\gamma\delta}]\ i\kappa V^{\nu\sigma\phi\theta\gamma\delta}\ \partial_{\lambda}\partial'_{\phi}\ i[{}_{\rho}\Delta_{\sigma}]\}$$

$$= -\kappa^2\Gamma^2(D/2-1)/16\pi^{D/2}\ (D-3)(D-2)^2D$$

$$\bullet\ \{(D+1)\eta^{\mu\nu}/\Delta x^{2D} - 2D\Delta x^{\mu}\Delta x^{\nu}/\Delta x^{2D+2}\}$$

$$= \text{Same} \bullet 1/[2(D-1)]\ [\eta^{\mu\nu}\partial^2 - \partial^{\mu}\partial^{\nu}]\ 1/\Delta x^{2D-2}$$

$[{}^{\mu}\Pi^{\nu}](x;x') = (\eta^{\mu\nu}\partial^2 - \partial^{\mu}\partial^{\nu})\,\Pi(x;x')$

- $\Pi_{3pt} = i\kappa^2\Gamma^2(D/2 - 1)/16\pi^D$
  $\bullet(D-3)(D-2)^2 D/2(D-1)\ 1/\Delta x^{2D-2}$
- $i[{}^{\mu}\Pi^{\nu}]_{4pt}$
  $= \partial_{\kappa}\{i\kappa^2 U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta}\ i[{}_{\alpha\beta}\Delta_{\gamma\delta}](x;x)\ \partial_{\lambda}\delta^D(x-x')\}$
  - Zero because $1/\Delta x^{D-2} \rightarrow 0$ for $x' = x$ in DR
- $i[{}^{\mu}\Pi^{\nu}]_{ctm} = 4iC_4(\eta^{\mu\nu}\partial^2 - \partial^{\mu}\partial^{\nu})\partial^2\delta^D(x-x')$
  - $\Pi_{ctm} = 4C_4\partial^2\delta^D(x-x')$

Renormalization

- Need $\int d^4x' \, \Pi(\Delta x^2)$ Regular(x')
- $1/\Delta x^{2D-2} = \frac{1}{2}(D-2)^{-2} \, \partial^2[1/\Delta x^{2D-4}]$

  $= \frac{1}{4}(D-2)^{-2}(D-3)^{-1}(D-4)^{-1} \, \partial^4[1/\Delta x^{2D-6}]$
- $0 = -\partial^2[1/\Delta x^{D-2}] + i4\pi^{D/2}/\Gamma(D/2-1) \, \delta^D(x-x')$
  - $1/\Delta x^{2D-6} - \mu^{D-4}/\Delta x^{D-2}$

    $= -\frac{1}{2}(D-4) \ln(\mu^2\Delta x^2)/\Delta x^2 + O[(D-4)^2]$
- Absorb delta function in C_4
  - $\Pi(\Delta x^2) = -i\kappa^2/192\pi^4 \, \partial^4[\ln(\mu^2\Delta x^2)/\Delta x^2] + O(\kappa^4)$

Schwinger-Keldysh Formalism (converting in-out to in-in)

- End of each line gets ± polarity
- Vertices all + (normal) or all – (-normal)
- Just replace Δx^2 in propagators
  - ++ ➔ $||x - x'||^2 - (|t - t'| - i\varepsilon)^2$
  - +- ➔ $||x - x'||^2 - (t - t' + i\varepsilon)^2$
  - -+ ➔ $||x - x'||^2 - (t - t' - i\varepsilon)^2$
  - -- ➔ $||x - x'||^2 - (|t - t'| + i\varepsilon)^2$
- In-out N-pt function gives 2^N in-in N-pt

SK Effective Field Equations

- $\Pi_{in\text{-}out} \rightarrow \Pi_{++} + \Pi_{+-}$
  - $t' > t$ ➔ ++ and +- cancel
  - $\Delta r > \Delta t$ ➔ ++ and +- cancel
  - $\Delta r < \Delta t$ ➔ $+- = -(++)^*$
- $\ln(\mu^2\Delta x^2)/\Delta x^2 = \frac{1}{8}\, \partial^2[\ln^2(\mu^2\Delta x^2) - 2\ln(\mu^2\Delta x^2)]$
  - $\ln(\mu^2\Delta x^2_{++}) - \ln(\mu^2\Delta x^2_{+-}) = i\pi\, \theta(\Delta t\text{-}\Delta r)$
  - $\ln^2(\mu^2\Delta x^2_{++}) - \ln^2(\mu^2\Delta x^2_{+-})$
    $= i2\pi\, \theta(\Delta t\text{-}\Delta r)\, \ln[\mu^2(\Delta t^2\text{-}\Delta r^2)]$
- $\Pi(x;x') = G/24\pi^2\, \partial^6\{\theta(\Delta t\text{-}\Delta r)$
  $[\ln[\mu^2(\Delta t^2 - \Delta r^2)] - 1]\} + O(G^2)$

No Change for Photons

- $\partial_\nu F^{\nu\mu}(x) + \int d^4x' \, \Pi(x;x')\partial'_\nu F^{\nu\mu}(x') = \mathcal{J}^\mu(x)$
- $\partial_\nu F^{\nu\mu}(x) = 0$ works for $\mathcal{J}^\mu = 0$
- Should be true to all orders

UV Enhancement for Coulomb

- $\Phi(r) = q/4\pi r\ [1 + 2G/3\pi r^2 + O(G^2)]$
  - $\#G/r^2$ fixed by dimensionality
  - No dependence on finite part of counterterm
- Same sign as S-matrix techniques
  - Radkowski (1970)
  - Bjerrum-Bohr (2002)
- Opposite to RG flow from β-function
  - Robinson & Wilczek (2006)
  - Harst & Reuter (2011)
  - But other things matter

Light-Cone Pushed Out

- Point dipole p^i created at t=0
  - $J^0(t,x\} = -\theta(t)\, p^i \partial_i\, \delta^3(x)$
  - $J^i(t,x\} = p^i\, \delta(t)\delta^3(x)$
- Induced $F_{ij}(t,x) = -\theta(t)\, (\partial_i p_j - \partial_j p_i)$ X
  - 0 Loop: $\delta(r^2 - t^2)/2\pi$
  - 1 Loop: $-4G/3\pi\, \delta'(r^2 - t^2)/2\pi$
- Hence $\delta(r^2 - t^2 - 4G/3\pi) + O(G^2)$
  - Super-luminal but Poincare invariant

Smearing of the Light-Cone

- An old idea (Pauli 1956, Deser 1957)
  - $i\Delta[g](x;x') = 1/\{2\pi^2\, \sigma[g](x;x')\} + O(\ln\sigma)$
  - $\int d^4x' \{i\Delta[g](x;x')\}^2$ doesn't exist
  - But $\int \Gamma[dg] \int d^4x' \{i\Delta[g](x;x')\}^2$ might
- Something quantum gravity should do
  - Light-cone is a quantum operator
  - S-luminal propagation thru fluctuations
- We might (eventually) build starships

Gauge Dependence

- 3-Parameter Poincare invariant gauge
  - $\mathcal{L}_{GRfix} = -1/2a\ \mathcal{F}^{\mu}\mathcal{F}_{\mu} \qquad \mathcal{F}_{\mu} = \partial^{\nu}h_{\mu\nu} - b/2\ \partial_{\mu}h$
  - $\mathcal{L}_{EMfix} = -1/2c\ (\partial^{\mu}A_{\mu})^2$
- $\Pi_{new}(x;x') = K(a,b)\ \Pi_{old}(x;x')$
  - $[F_{new}]_{\mu\nu}(x) = K(a,b)\ [F_{old}]_{\mu\nu}(x)$
  - $K = -3a/2\ (3b^2-12b+8)/(b-2)^2 - 1/2(b-2)^2$
- Dynamical gravitons contribute nothing!

What Gives????

- Need 1 loop correction to J^{μ}
- Must also fix "t" and "r" geometrically
  - Also frames in which $F^{\mu\nu}$ reported
- But our results agree with those from gauge independent S-matrix

Summary

- $\partial_\nu F^{\nu\mu}(x) + \int d^4x' \, \Pi(x;x')\partial'_\nu F^{\nu\mu}(x') = J^\mu(x)$
  - $\Pi = G/(24\pi^2)\, \partial^6 \{\theta(\Delta t-\Delta r)\, [\ln[\mu^2(\Delta t^2-\Delta r^2)] - 1]\} + O(G^2)$
- Dynamical photons unchanged
- $\Phi(r) \rightarrow q/(4\pi r)\, [1 + 2G/(3\pi r^2) + O(G^2)]$
- Pulse $\rightarrow r^2 = t^2 + 4G/3\pi + O(G^2)$
  - NB Poincare invariance inevitable in pert.
- Gauge dependence requires work
  - But S-matrix results gauge independent

Overlaps with LQG

- SK field eqns can be used like Maxwell
  - Finite space & time, not just scattering
  - Essential for cosmology!
- Releasing $\Psi_{L,R}$ at t=0 ➔ need int. state
  - But only perturbatively to some order
- Gauge dependence ➔ inv. Observables
  - But again, only perturbatively
  - And can use initial value surface
- Perhaps we can converge?