Black hole collapse and bounce in effective loop quantum gravity

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Work with Jarod G. Kelly and Robert Santacruz
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Motivation

Black holes are one of the most promising places to explore, and potentially even to test, a theory of quantum gravity.

- In classical general relativity, a black hole contains a curvature singularity,
- The information loss problem: according to QFT on a curved background, a black hole created by infalling matter in a pure state appears to act as a black body and evaporate to a mixed state of Hawking radiation, [Hawking, 1975]
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Our goal in this work is to use LQG and LQC techniques to study quantum gravity effects in black holes, and to try to make some progress on these problems.

General Framework

We will study spherically symmetric black hole space-times by:

1. imposing spherical symmetry at the classical level (constraints, symplectic structure), [Bojowald, Swiderski, 2006]
2. simplifying the problem by gauge-fixing the diffeomorphism constraint, again at the classical level, [Campiglia, Gambini, Pullin, 2007]
3. including holonomy corrections in an effective framework, [Modesto, 2004; Ashtekar, Bojowald, 2006; Boehmer, Vandersloot, 2007; Chiou, Ni, Tang, 2012; Gambini, Olmedo, Pullin, 2020; . . .]
4. finding and solving the effective equations of motion.

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4. finding and solving the effective equations of motion.

We start with vacuum space-times, but the resulting space-time is incomplete.

So we then add a dust field [see also Bojowald, Reyes, Tibrewala, 2009; ...] to be able to describe the collapse and the entire black hole interior.

Variables and Gauge-Fixed Hamiltonian

The general spherically symmetric metric is

$$ds^2 = -N(x,t)^2 \mathrm{d}t^2 + f(x,t)\,\left[\mathrm{d}x + N^x(x,t)\mathrm{d}t\right]^2 + g(x,t)\,\mathrm{d}\Omega^2.$$

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Here $b \sim K_\theta^2, K_\phi^3$ captures the extrinsic curvature in the angular directions, and its conjugate variable is $E^b = \sqrt{fg}$.

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There remains only the scalar constraint,

$$\mathcal{H} = \frac{1}{2G\gamma}\left[\frac{3\gamma x}{E^b} - \frac{2\gamma x^2}{(E^b)^2}\partial_x E^b - \frac{E^b}{\gamma x}\partial_x\left[x(b^2+\gamma^2)\right]\right] \approx 0.$$

The dynamics are generated by $\mathcal{C} = \int N\mathcal{H}$.

Holonomy Corrections

Now we want to include holonomy corrections. To do this, we express b in terms of its parallel transport along edges of a minimal length $\sqrt{\Delta} \sim \ell_{\rm Pl}$. This has to be done in $\mathcal{H}$, and also for the relation between N and N^x.

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An important point is that the Planckian length of the edge is a physical length, not a coordinate length (this is the basis of the 'improved' or $\bar{\mu}$ dynamics [Ashtekar, Pawłowski, Singh, 2005]). For an edge in the θ direction, the metric gives ${\rm d}s = x\,{\rm d}\theta$ so a physical length $\delta s = \sqrt{\Delta}$ implies $\delta\theta = \sqrt{\Delta}/x$.

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- This is based on the LQC 'K' loop quantization [Vandersloot, 2007; Singh, WE, 2014].
- The path-ordered exponential trivializes due to spherical symmetry. (It would not trivialize for an edge in the radial direction, but this has been avoided by the gauge-fixing.)

Aside: Comments on the $\bar{\mu}$ Scheme

This specific $\bar{\mu}$ scheme was first proposed by Boehmer and Vandersloot (2007) for the Kantowski-Sachs space-time (which describes the black hole interior in GR). But this choice gave unacceptably large effects at the horizon.

We believe that this problem is due to the use of coordinates that become null at the horizon: the physical length of a short path in the radial direction goes to 0, but the $\bar{\mu}$ scheme requires imposing that this physical length be $\sim \ell_{\rm Pl}$. This tension could be the source of large QG effects at the horizon.

Aside: Comments on the $\bar{\mu}$ Scheme (Continued)

The problem of null coordinates can be avoided by using horizon-piercing coordinates, as first tried by Chiou, Ni and Tang (2012), and more recently by Gambini, Olmedo and Pullin (2020) and us. Note that the constraints obtained by Chiou, Ni and Tang did not have a closed algebra; this seems to be due to the use of a 'point holonomy' approximation in the radial direction to avoid evaluating the path-ordered exponential.

With this $\bar{\mu}$ scheme applied to the full space-time (not restricting to Kantowski-Sachs), there appear to be 3 ways to get a closed constraint algebra:

1. avoid using the 'point holonomies' approximation and instead evaluate the path-ordered exponential in x (seems hard),
2. try a different combination of the constraints [Gambini, Olmedo, Pullin, 2020], but hard to include matter,
3. impose a gauge (our approach).

Effective Scalar Constraint

This process gives the effective scalar constraint

$$\mathcal{H}^{(eff)} = \frac{1}{2G\gamma}\left[\frac{3\gamma x}{E^b} - \frac{2\gamma x^2}{(E^b)^2}\,\partial_x E^b - \frac{E^b}{\gamma x}\partial_x\left(\frac{x^3}{\Delta}\sin^2\frac{\sqrt{\Delta}\,b}{x} + \gamma^2 x\right)\right],$$

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Note that even after fixing the areal gauge, there is a non-trivial constraint algebra:

$$\{\mathcal{C}[N_1], \mathcal{C}[N_2]\} = C\left[-\frac{x}{\gamma\sqrt{\Delta}}\sin\frac{\sqrt{\Delta}\,b}{x}\cos\frac{\sqrt{\Delta}\,b}{x}\,(N_1\partial_x N_2 - N_2\partial_x N_1)\right].$$

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We also need to update the relation between N and N^x. Setting $N^x = -N\cdot\frac{x}{\gamma\sqrt{\Delta}}\sin\frac{\sqrt{\Delta}\,b}{x}\cos\frac{\sqrt{\Delta}\,b}{x}$ gives a constraint algebra that looks the same as in classical GR, [This N^x also agrees with Gambini, Olmedo, Pullin, 2020]

$$\{\mathcal{C}[N_1], \mathcal{C}[N_2]\} = C\left[N_1^x\partial_x N_2 - N_2^x\partial_x N_1\right].$$

Results

Looking for a stationary solution, the effective dynamics can be solved explicitly for Painlevé-Gullstrand-like coordinates with $N = 1$:

$$\mathrm{d}s^2 = -\left(1 - \frac{R_S}{x} + \frac{\gamma^2 \Delta R_S^2}{x^4}\right) \mathrm{d}t^2 + 2\sqrt{\frac{R_S}{x}\left(1 - \frac{\gamma^2 \Delta R_S}{x^3}\right)}\, \mathrm{d}t\, \mathrm{d}x + \mathrm{d}x^2 + x^2 \mathrm{d}\Omega^2,$$

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It is of course possible to make different choices for the lapse. For example, for $N = 1/(\sqrt{1 + R_S/x})$ we recover the effective space-time found by Gambini, Olmedo and Pullin (2020).

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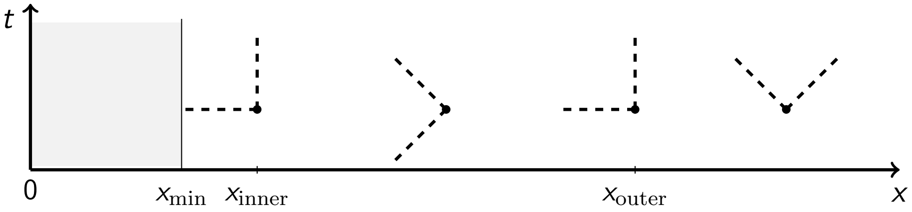
An intriguing result is that for $M \lesssim m_{\rm Pl}$, there is no horizon: a mass greater than $m_{\rm Pl}$ is needed to create a black hole.

Properties of the Solution

For $M \gg m_{\rm Pl}$, there is an outer horizon at $x_{\rm outer} \approx R_S - \frac{\gamma^2 \Delta}{R_S}$ and an inner horizon at $x_{\rm inner} \approx x_{\rm min} + (\gamma^4 \Delta^2 / 27 R_S)^{1/3}$:



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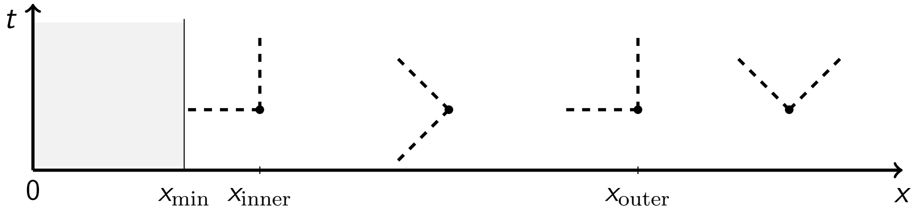

Also, curvature invariants are bounded above by:

$$\lim_{x\to x_{\rm min}} R = -\frac{6}{\gamma^2\Delta}, \qquad \lim_{x\to x_{\rm min}} R_{\mu\nu}R^{\mu\nu} = \frac{90}{\gamma^4\Delta^2},$$

$$\lim_{x\to x_{\rm min}} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{360}{\gamma^4\Delta^2}.$$

Adding Matter

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To generate a gravitational field, there must be a source and in spherical symmetry there are no gravitational waves, so we need mass. Classically, it is possible to have a point mass at the origin, $\rho(x) = M\,\delta(x)$.

But if quantum gravity imposes an upper bound on $\rho \leq \rho_c \sim \rho_{\mathrm{Pl}}$, then a compact object of mass M must extend to a radius of at least $x_{\mathrm{min}} \sim (M/\rho_c)^{1/3}$.

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To explore this, we add a zero-pressure dust field. We use the dust-time gauge [Husain, Pawłowski, 2012], implying $N = 1$, in addition to the areal gauge; the resulting equations of motion are LQG effective equations for Lemaître-Tolman-Bondi (LTB) space-times in Painlevé-Gullstrand-like coordinates.

A nice property of the dust-time gauge is that the physical Hamiltonian is algebraically identical to the gravitational part of the constraint, so

$$H_d^{(eff)} = \frac{1}{2G\gamma}\left[\frac{3\gamma x}{E^b} - \frac{2\gamma x^2}{(E^b)^2}\,\partial_x E^b - \frac{E^b}{\gamma x}\partial_x\left(\frac{x^3}{\Delta}\sin^2\frac{\sqrt{\Delta}\,b}{x} + \gamma^2 x\right)\right].$$

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The effective equations of motion for LTB space-times are

$$\dot{E}^b = -\frac{x^2}{2\gamma\sqrt{\Delta}}\partial_x\left(\frac{E^b}{x}\right)\sin\frac{\sqrt{\Delta}\,b}{x}\cos\frac{\sqrt{\Delta}\,b}{x},$$

$$\dot{b} = \frac{\gamma}{2}\left(\frac{x}{(E^b)^2} - \frac{1}{x}\right) - \frac{1}{2\gamma\Delta x}\partial_x\left(x^3\sin^2\frac{\sqrt{\Delta}\,b}{x}\right).$$

Oppenheimer-Snyder Collapse

Let's consider the simple Oppenheimer-Snyder collapse model: a star of radius $L(t)$, with $\rho_d = 0$ outside the star and $\rho_d = \rho(t)$ inside the star.

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If we neglect edge effects at $x = L(t)$, we can derive simple equations of motion for this system from the effective dynamics for general LTB space-times:

$$\left(\frac{\dot{L}}{L}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \rho = \frac{3M}{4\pi L^3}.$$

In this context, the LTB effective dynamics imply the LQC effective Friedman equation for flat FLRW space-times, and even ρ_c is exactly the same as in LQC. We can also find explicit solutions for E^b, b and N^x: there is a bounce.

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Further, the minimum value of $L(t)$ is exactly equal to $x_{\rm min}$. So, the smallest radius x for which the vacuum solution needs to exist is precisely $x_{\rm min}$: exactly where the vacuum solution ends.

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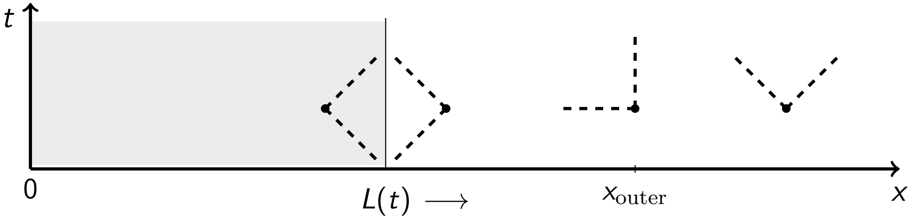
But this is not exactly a white hole: the region just inside the outer horizon is still trapped; it is only within the dust matter field that there is an anti-trapped region.



As the system evolves, the edge L moves outwards. Once L passes x_{outer}, there will not be a black hole anymore.

Outgoing Shock Wave

The Oppenheimer-Snyder solution cannot be trusted after the bounce since it neglects edge effects. Of course, edge effects become important due to the discontinuity at L.

This discontinuity is a shock wave in the gravitational field itself—specifically, $b(x)$ is discontinuous. To understand how L moves outwards it is necessary to use the full LTB effective equations.

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A back of the envelope estimation suggests the discontinuity in b slows the expansion, and the lifetime of the black hole could be $\sim GM^2/m_{\rm Pl}$. Further work is needed to verify this estimate.

Note: here the lifetime of the black hole corresponds to the time elapsed between the Killing horizon being formed by the collapsing star, and then the front L of the shock wave expanding past $x_{\rm outer}$, as measured by a distant observer.

Ramifications for the Information Loss Problem

If the lifetime of a black hole is indeed $\sim GM^2/m_{\rm Pl}$, then this could have interesting ramifications for the information loss problem.

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Further, it is often argued that potential information loss becomes a problem once half of the black hole has evaporated [Page, 1993]. The entanglement entropy between the Hawking radiation and the black hole can increase until the Page time, when there are more degrees of freedom in the Hawking radiation than are left in the black hole—at this point purification must occur for the entanglement entropy to decrease. The Page time for a black hole is $\sim GM^3/m_{\rm Pl}^2$.

But if the black hole has a lifetime of $\sim GM^2/m_{\rm Pl}$, then Hawking evaporation is a subleading quantum effect compared to the bounce. There is not enough time for information loss to become a problem.

Summary

- We derive LQG effective equations with holonomy corrections for spherical symmetry in vacuum and with dust.
- In vacuum, we find a stationary solution that agrees with Gambini, Olmedo and Pullin (2020). There is an inner horizon, and minimum radius where the vacuum solution breaks down.
- By adding a dust matter field, we can describe the entire space-time. For the Oppenheimer-Snyder model of black hole collapse, there is a non-singular bounce from the collapsing matter to an expanding 'white hole' shock wave solution.
- We estimate the lifetime of the black hole to be $\sim GM^2/m_{\rm Pl}$, in which case the information loss problem would be avoided.

Open Questions

- How should the $\bar{\mu}$ scheme be implemented if a coordinate becomes null?
- The (reduced) constraint algebra is deformed. What are the physical ramifications of this?
- Determine the dynamics of the white hole shock wave more exactly, and calculate the lifetime of a black hole with more precision.
- Explore other forms of black hole collapse beyond the simplest Oppenheimer-Snyder model.
- Couple other types of matter. What is the role of pressure?

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Thank you for listening!