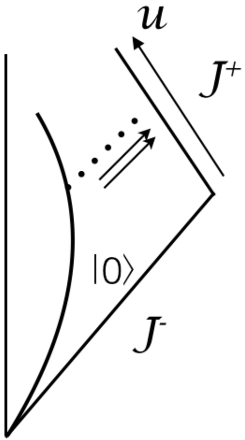
Black holes and reversibility

Matteo Smerlak

Perimeter Institute for Theoretical Physics

ILQGS
March 24, 2015

Setup: gravitational collapse



- A black hole forms from ingoing matter.
- Trapping horizon forms and peels off outgoing geodesics.
- Thermal Hawking radiation is emitted.
- Breakdown of predictability?

Information loss as a physical problem

"*Information loss violates a basic tenet of quantum mechanics.*"

- Information loss happens all the time:
  - with open systems (decoherence)
  - with non-Cauchy "out" surfaces

[Wald 13]

- Information loss does not mess up with conservation laws.

[Banks, Peskin, Susskind 84; Unruh, Wald 95]

- The only real question is: what difference would it make?

What physical effects relate to the information loss problem?

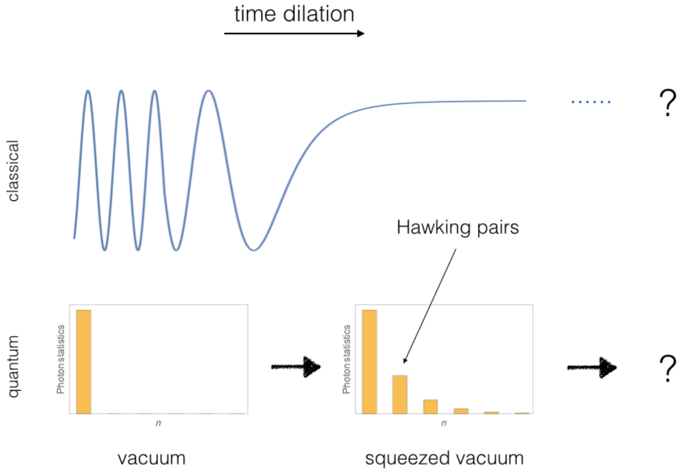
Outline

Black holes as squeezers

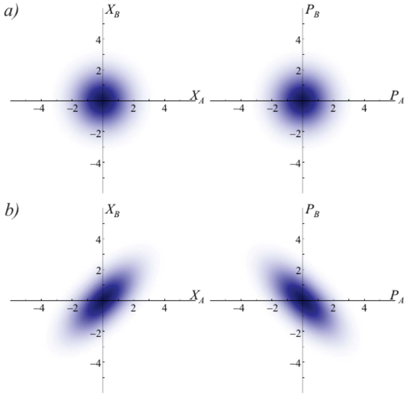
Past/future entanglement

(A)cyclic processes

The Hawking effect



Two-mode squeezed vacuum



$$|\psi_{AB}\rangle \propto \sum_{n=0}^{\infty} (\tanh r)^n |n, n\rangle \quad \Longrightarrow \quad \rho_A \propto \sum_{n=0}^{\infty} (\tanh r)^n |n\rangle\langle n|$$

Stronger squeezing, higher temperature ($e^{-\hbar\omega/kT} = \tanh r$).

Observing TMSV

LETTER
doi:10.1038/nature10561

Observation of the dynamical Casimir effect in a superconducting circuit

C. M. Wilson[1], G. Johansson[1], A. Pourkabirian[1], M. Simoen[1], J. R. Johansson[2], T. Duty[3], F. Nori[2,4] & P. Delsing[1]

SQUID

ARTICLES
PUBLISHED ONLINE: 12 OCTOBER 2014 | DOI: 10.1038/NPHYS3104
nature physics

Observation of self-amplifying Hawking radiation in an analogue black-hole laser

Jeff Steinhauer

BEC

Selected for a Viewpoint in *Physics*
PRL **105**, 203901 (2010) PHYSICAL REVIEW LETTERS week ending 12 NOVEMBER 2010

Hawking Radiation from Ultrashort Laser Pulse Filaments

F. Belgiorno,[1] S. L. Cacciatori,[2,3] M. Clerici,[3] V. Gorini,[2,3] G. Ortenzi,[4] L. Rizzi,[3] E. Rubino,[3] V. G. Sala,[3] and D. Faccio[3,5,*]

Nonlinear optics

PRL **106**, 021302 (2011) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2011

Measurement of Stimulated Hawking Emission in an Analogue System

Silke Weinfurtner,[1] Edmund W. Tedford,[2] Matthew C. J. Penrice,[1] William G. Unruh,[1] and Gregory A. Lawrence[2]

Hydrodynamics

Open questions

The evaporation problem is a runaway problem

$$\text{radiation} \implies \text{mass loss} \implies \text{smaller hole} \implies \text{higher squeezing} \implies \text{more radiation...}$$

The questions for us are

- does this lead to an explosive behavior?
- does thermality break down at late times?
- what astrophysical signatures should we look for?

Outline

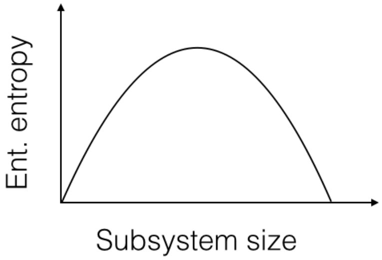
Entanglement in finite systems

In finite dimensions, entanglement entropy

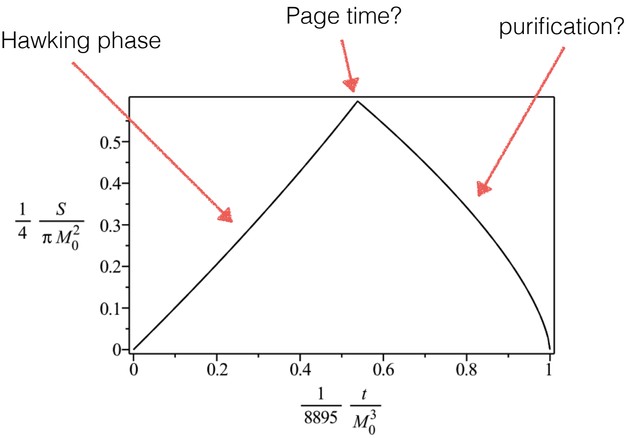
$$S[\rho_A] \equiv -\mathrm{tr}_A[\rho_A \ln \rho_A] \quad \text{with} \quad \rho_A \equiv \mathrm{tr}_B[\rho_{AB}]$$

is unitarily invariant and satisfies the triangle inequality

$$|S[\rho_A] - S[\rho_B]| \leq S[\rho_{AB}] \leq S[\rho_A] + S[\rho_B].$$



Page's conjecture



[Page (93,13)]

Reversibility : three open questions

Is the evaporation process

1. unitary, viz. is purity preserved ?

$$S_{\mathrm{vN}}[\rho_{\mathrm{out}}] = S_{\mathrm{vN}}[\rho_{\mathrm{in}}] \ ?$$

[Hawking (76)]

2. cyclic, viz. does entanglement return to its initial value?

$$\lim_{u \to +\infty} S_{\mathrm{P}}(u) = \lim_{u \to -\infty} S_{\mathrm{P}}(u) \ ?$$

[Page (93)]

3. conservative, viz. do energy input and output match ?

$$\lim_{u \to +\infty} M(u) = 0 \ ?$$

Working assumptions

Neglect

- angular momentum (of spacetime and fields)
- backscattering
- non-conformal interactions

but not

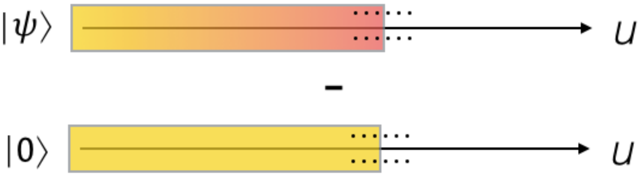
- semiclassical backreaction (even strong).

Reduces field dynamics to 2d CFT:

$$\phi(t,r) = r^2 \int_{S^2} d\Omega^2 \, \Phi(t,r,\Omega)$$

Renormalized entanglement entropy

In QFT, entanglement entropy is UV-divergent. Substract vacuum contribution



Defines renormalized entanglement entropy

$$S_{\mathrm{P}}(u) = [\rho_\psi(u)] - S[\rho_0(u)]$$

[Holzhey, Larsen, Wilczek (94)]

The Page curve

Starting from the (non-covariant) CFT formula for a segment

$$S[\rho(R)] = \frac{1}{3} \log \frac{L(R)}{\epsilon}$$

[Holzhey, Larsen, Wilczek (94)]

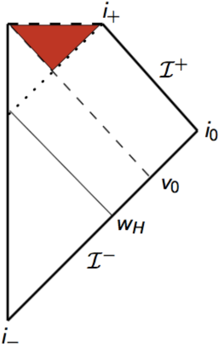
we obtain the geometric formula

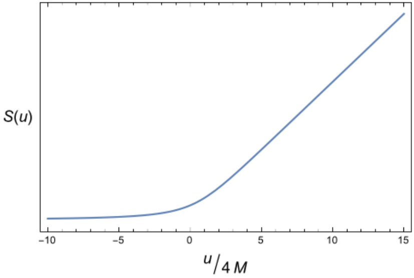
$$S(u) = \frac{1}{12} \ln \chi(u)$$

[Bianchi, MS 14]

with $\chi = \omega_+/\omega_-$ the in-out redshift factor.

Vaidya spacetime: the Hawking phase

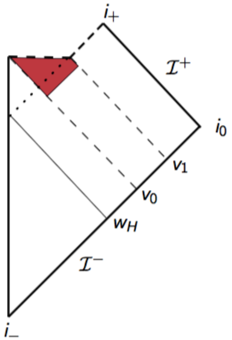


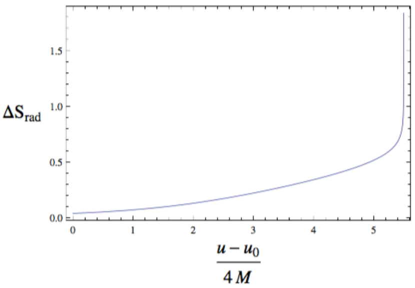


$$S(u) = \frac{1}{12} \log \left(\frac{1 + W\left(e^{-u/4M}\right)}{W\left(e^{-u/4M}\right)} \right) \sim \frac{u}{48M}$$

[Bianchi, de Lorenzo, MS 14]

"Hawking spacetime": thunderbolt





$$S(u) \sim \frac{1}{12} \log\left(\frac{4M}{u-u_H}\right)$$

[Bianchi, de Lorenzo, MS 14]

From spacetime to the Page curve

More examples illustrate the connection between geometry and entanglement...

[Bianchi, de Lorenzo, MS 14]

... but in this approach, where

$$\text{spacetime} \implies \text{entropy},$$

backreaction is an input. Next best thing after blind guess!

Importance of other, less narrow approach.

Outline

Black holes as squeezers

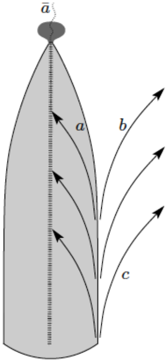
Past/future entanglement

(A)cyclic processes

Unitarity violations?

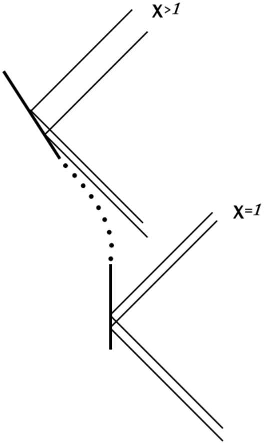
Several authors propose that evaporation is non-unitary (in the QFT sector):

- decoherence without dissipation: spin bath model
  [Unruh, Wald 95; Unruh 12]
- quantum gravity decoherence: defects in spacetime weave
  [Perez 14]



Here I'll explore another possibility: unitary but acyclic evaporation.

The moving mirror



- Mirror starts at rest...
- ... then accelerates...
- ... then is inertial again.

$$\Delta S \propto \text{(relative rapidity)}$$

Unitary but acyclic.

What does cyclicity imply?

Outgoing energy flux

Other natural observable at $\mathcal{I}^+$: energy flux

$$F(u) \equiv 4\pi r^2 \langle \mathrm{in} | T_{uu} | \mathrm{in} \rangle$$

and Bondi mass

$$M(u) \equiv M_0 - \int_{-\infty}^{u} du' \, F(u').$$

In the 2d approximation,

$$F(u) = -\frac{1}{24\pi} \left(\frac{\dddot{p}(u)}{\dot{p}(u)} - \frac{3}{2} \frac{\ddot{p}(u)^2}{\dot{p}(u)^2} \right)$$

[Fulling, Davies, Unruh (76)]

The it from bit equation

$$\boxed{2\pi F(u) = 6\dot{S}(u)^2 + \ddot{S}(u)}$$

- "Page curve" $S(u)$ determines energy flux $F(u)$
- Energy flux $F(u)$ determines Page curve $S(u)$, via

$$-\ddot{\psi}(u) + 12\pi F(u)\,\psi(u) = 0 \quad \text{where} \quad \psi \equiv e^{6S}$$

- Flux $F(u)$ is "exceptional": $F(u) + \delta F(u)$ not a flux
- Implies quantum inequality: $|F|\tau^2 \lesssim 1$

It-from-bit and the GSL

$$\boxed{2\pi F(u) = 6\dot{S}(u)^2 + \ddot{S}(u)}$$

Generalizes GSL in two ways:

- Includes non-adiabatic term (identity rather than ineq.)
- Does not require special causal structure (event horizon)
- Gives back GSL when $|\ddot{S}| \ll \dot{S}^2$. For a Schwarzschild black hole, with

$$\dot{S} = \frac{1}{48M_B} \quad \text{and} \quad F = -\dot{M}_B = -\frac{\dot{S}_{\rm BH}}{32\pi M_B}$$

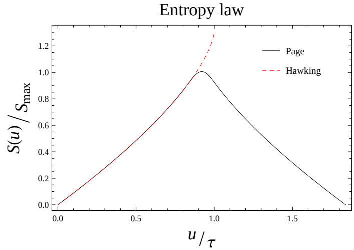
you get

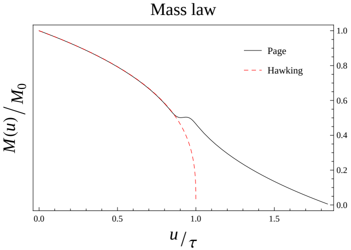
$$dS_{\rm BH} + dS = \frac{u}{96M} > 0.$$

A black hole's last gasp

$$\boxed{2\pi F(u) = 6\dot{S}(u)^2 + \ddot{S}(u)}$$

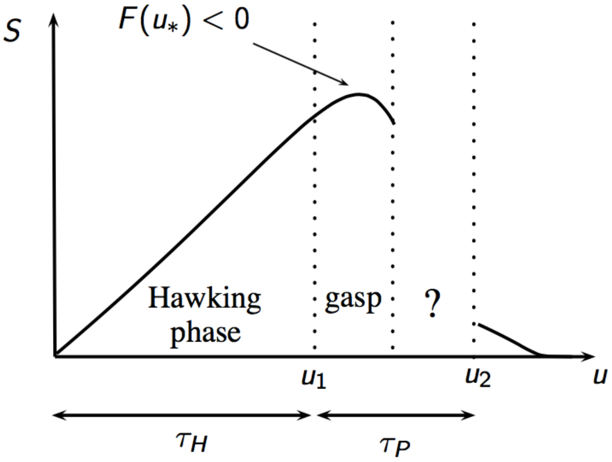
At the "Page time" u_*, the flux is negative: $F(u_*) < 0$.





Black hole's "last gasp".

Time scales



Lifetime of a black hole

From the it-from-bit equation we get that if

- the evaporation process is cyclic
- energy is conserved: $M_B(u) > 0$,

then the purification time must be large:

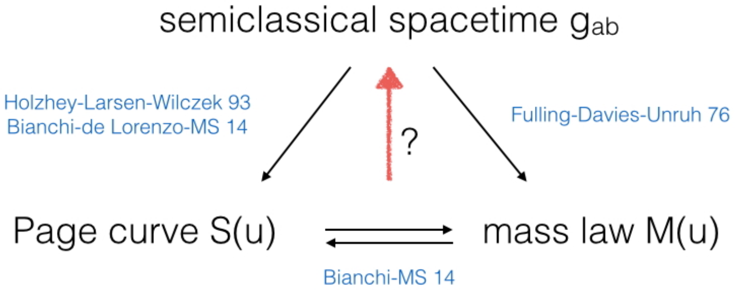
$$\tau_P \geq \xi \, \frac{(M_0^2 - M_1^2)^2}{M_1 m_P^2} = \begin{cases} \mathcal{O}(M_0^4/m_P^3) & \text{if} \quad M_1 = \mathcal{O}(m_P) \\ \mathcal{O}(M_0^3/m_P^2) & \text{if} \quad M_1 = \mathcal{O}(M_0/2) \end{cases}$$

[Carlitz-Willey 87; Bianchi, MS 19]

Recent nonsingular black hole models fail to respect this bound.

[Frolov, Vilkoviski 91; Hayward 06; Bardeen 14; Rovelli, Vidotto, Haggard 14]

An open problem



1. Is there a nonsingular black hole spacetime such that evaporation is cyclic and (sub)-conservative?
2. What kind of spacetime does Page's curve describe?

Conclusions

- Focus on asymptotic observers (us).
- In field theory, unitarity is not equivalent to cyclicity.
- From a (guessed) geometry, can compute the Page curve.
- Inverse problem seems insightful, thanks to it-from-bit.

Thanks to
A. Ashtekar, E. Bianchi, T. de Lorenzo, A. Perez, C. Rovelli.