Quantum theory of charged black hole horizons

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K.E and H.S: Phys. Rev. D **97**, 086016

December 3, 2018

Section 1

Introduction

Introduction

In this talk:

- Loop quantum gravity
- Quantized isolated horizons
- Yang-Mills matter fields

Why?

- Matter contribution to BH entropy?
- Consistency check of LQG

Introduction

Possible matter contribution to BH entropy:

Entanglement entropy of matter [Bombelli et al. 86, Srednicki '93]

$$S_{\text{ent}} = c\,\frac{A}{l_{\text{UV}}^2} + \dots$$

Log corrections from Euclidean path integral [Sen '12]

For example: Electromagnetic field

$$S_{\text{PI}} = \frac{a_H}{4} + \left(C_1 + (4\pi)^3 C_2 \frac{Q_H^4}{a_H^2} + \dots \right) \ln a_H + \dots$$

In the context of LQG: Indistinguishable punctures with matter degeneracy [Ghosh, Noui, Perez '13].

Introduction

In LQG:

- kinematical quantization for matter fields
- particularly natural: YM fields
- not used for quantum IH (notable exception: non-minimally coupled scalar [Ashtekar, Corichi, Sudarsky '03])

By including YM matter fields: Consistency check.

- implementation of matter boundary conditions on the horizon
- Bekenstein-Hawking law for the entropy?
- U(1) vs. SU(2) gauge fixing

In addition:

- also count YM states on the horizon?

Section 2

Diffeo covariant quantum Yang Mills theory

Hamiltonian YM theory

Consider YM theory with simple compact gauge group G:

YM action

$$S_{\mathrm{YM}}[A] = \frac{1}{8\pi g^2} \int_M \langle F \wedge *F \rangle = \frac{1}{16\pi g^2} \int_M \sqrt{-g} F^{\mu\nu I} F_{\mu\nu I} \, \mathrm{d}^4 x$$

- $\langle \cdot, \cdot \rangle$: minus the Killing metric of $\mathrm{Lie}(G)$ (pos. definite !)
- vector potential: $\mathbf{A} = i_\Sigma^* A$
- YM magnetic and electric fields: $\mathbf{B} = i_\Sigma^* F$, $\mathbf{E} = i_\Sigma^*(*F)$

Diffeomorphism covariant quantization

Poisson bracket

$$\{\mathbf{A}_a^I(x), \mathbf{E}_J^b(y)\} = -4\pi g^2 \delta_a^b \delta_J^I \delta^{(3)}(x, y)$$

Basic algebra generated by holonomies and fluxes [Rovelli,Smolin,Ashtekar+Lewandowski,...]

$$\mathbf{h}_p(\mathbf{A}) = \mathcal{P} \exp\left(\int_p \mathbf{A}\right) \quad \mathbf{E}_n(S) = \frac{1}{g} \int_S n^I \mathbf{E}_I$$

Diffeomorphism covariant quantization

Quasilocal definition of non-Abelian charge subtle.

For S closed oriented surface

Modulus of flux [Corichi+Nucamendi+Sudarsky'00,Ashtekar+Beetle+Fairhurst'00]

$$\text{electric: } Q_S := \frac{1}{4\pi}\int_S \|\mathbf{E}\| \quad \text{magnetic : } P_S = \frac{1}{4\pi}\int_S \|\mathbf{B}\|$$

Covariant flux [Abbot, Deser '82, Thiemann'00, Zilker+S, E+S]

$$Q_S'^{\,I} := \frac{1}{4\pi}\int_S \mathrm{Ad}_{\mathbf{h}}(\mathbf{E})^I \qquad P_S'^{\,I} = \frac{1}{4\pi}\int_S \mathrm{Ad}_{\mathbf{h}}(\mathbf{B})^I$$

Diffeomorphism covariant quantization

Quantization as for gravity, with gauge group G: [Thiemann '97]

$$\mathcal{H}_{YM} = L^2(\bar{\mathcal{A}}_G, \mathrm{d}\mu^G_{\mathrm{AL}})$$

Basis: Charge-network states $|\gamma, \underline{\lambda}\rangle$, λ_e: highest weight labels

Charge operator ($\leftrightarrow$ area operator in LQG)

$$\widehat{Q}(S)\,|\gamma, \underline{\lambda}\rangle = g\left(\sum_{p\in\gamma\cap S} Q(\lambda_p)\right)|\gamma, \underline{\lambda}\rangle$$

$\widehat{Q'}(S)$ more complicated. Operates in $\bigotimes_p \pi_{\lambda_p}$.

Diffeomorphism covariant quantization

Electrodynamics: special case $G = \mathrm{U}(1)$.

- Basis: Charge-network states $|\gamma, \underline{n}\rangle = h_{e_1}^{n_1} \cdots h_{e_k}^{n_k}$
- $n_e \in \mathbb{Z}$ (charges)
- For closed surfaces S: Gauss law $\Rightarrow$ charge operator $\widehat{Q'}(S) := -\frac{1}{4\pi}\hat{\mathbf{E}}(S)$

Action of charge operator [Corichi+Krasnov'97]

$$\widehat{Q'}(S)\,|\gamma, \underline{n}\rangle = g\left(\sum_{e\cap S\neq\emptyset} n_e\right)|\gamma, \underline{n}\rangle$$

g: elementary charge

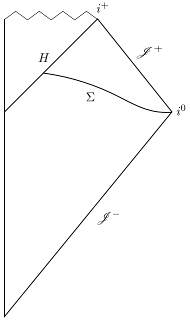
Section 3

Distorted charged black holes

Classical phase space

Black hole in the presence of gauge matter fields

- M has inner boundary given by IH $H \cong \mathbb{R} \times S^2$
- $\Rightarrow$ YM action picks up boundary term
- $\Rightarrow$ can be neglected by appropriately restricting the phase space [C+N+S '00,A+B+F '00]
- IH requires $T_{\mu\nu} l^\mu l^\nu \hat{=} 0$ along any null normal l^μ



Energy-momentum tensor YM-field

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\rm YM}}{\delta g^{\mu\nu}} = \frac{1}{4\pi} \left(\langle F_{\mu\rho}, F_\nu{}^\rho \rangle - \frac{1}{4} g_{\mu\nu} \langle F_{\alpha\beta}, F^{\alpha\beta} \rangle \right)$$

Classical phase space

Use spin structure $\mathrm{Spin}^+(M)$ to work out symmetries

- $F_{\mu\nu}$ antisymmetric $\Rightarrow F_{AA'BB'}{}^I = \phi_{AB}{}^I \bar{\epsilon}_{A'B'} + \bar{\phi}_{A'B'}{}^I \epsilon_{AB}$
  $\phi_{(AB)}{}^I$: $\mathfrak{g}_{\mathbb{C}}$-valued symmetric tensor

EM-tensor in spinor bundle

$$T_{AA'BB'} = -\frac{1}{2\pi}\langle \phi_{AB}, \phi_{AB}\rangle =: -\frac{1}{2\pi}\|\phi_{AB}\|^2$$

- use $T_{\mu\nu} l^\mu l^\nu \hat{=} 0$

Matter BC (NEHs)

$$\underset{\Leftarrow}{\mathbf{E}}{}^I = -2\mathrm{Re}(\phi_1^I)\,\mathrm{vol}_{S^2} \qquad \underset{\Leftarrow}{\mathbf{B}}{}^I = -2\mathrm{Im}(\phi_1^I)\,\mathrm{vol}_{S^2}$$

- generalization of [Ashtekar+Corichi+Krasnov '99, Corichi+Nucamendi+Sudarsky '00]
- ϕ_1^I $\mathfrak{g}_{\mathbb{C}}$-valued Newman-Penrose coefficient ($\phi_1^I := \iota^A o^B \phi^I_{AB}$)

Classical phase space

Extending [Engle, Noui, Perez, Pranzetti '10] [Perez, Pranzetti '10]:

total action

$$S = S_{\mathrm{grav}}[A, E] + S_{\mathrm{YM}}[\mathbf{A}, \mathbf{E}] + S_{\mathrm{CS}}[A_{\sigma_+}] + S_{\mathrm{CS}}[A_{\sigma_-}]$$

Symplectic structure $\mathrm{SU}(2)$ Chern-Simons theory: ($k_\pm$ CS level)

$$\Omega^{\pm}_{\mathrm{CS}}(\delta_1, \delta_2) = \frac{k_\pm}{4\pi} \int_{S^2} \delta_1 A^i_{\sigma_\pm} \wedge \delta_2 A_{\sigma_\pm\, i}$$

where $A^i_{\sigma_\pm} := \Gamma^i + \sqrt{\frac{2\pi}{a_H}}\sigma_\pm e^i$ $\mathrm{SU}(2)$ connections on S^2 with curvature

$$F(A_{\sigma_\pm})^i = F(\Gamma)^i + \frac{2\pi}{a_H}\sigma^2_\pm (*E)^i + \mathrm{d}_\Gamma e^i.$$

Classical phase space

BC in terms of Ashtekar connection: [A+C+K '99]

$$F(\underleftarrow{A^+})^i = 2\left(\Psi_2 - \Phi_{11} - \tfrac{R}{24}\right)(\underleftarrow{*E})^i$$

can be rewritten using

- $\mathrm{d}_\Gamma e^i = 0$ (Γ torsion-free)
- $F(\Gamma)^i = F(A^+)^i + c(*E)^i$ ($c: H \to \mathbb{R}$ extrinsic curvature scalar)
- $\Phi_{11} = 2\pi T_{\mu\nu}(l^\mu k^\nu + m^\mu \bar{m}^\nu) = 2\|\phi_1\|^2$ for YM-fields

Coupling bulk $\leftrightarrow$ horizon (incl. matter):

$$F(A_{\sigma_\pm})^i = 2\left(\Psi_2 - 2\|\phi_1\|^2 + \tfrac{\pi}{a_H}\sigma_\pm^2 + \tfrac{c}{2}\right)\underleftarrow{*E}^i$$

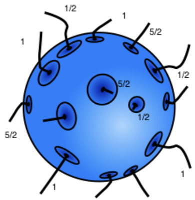
Subsection 1

Quantum theory

Quantum theory

Extending [Engle, Noui, Perez, Pranzetti '10] [Perez, Pranzetti '10]:

Hilbert space: $\mathcal{H}_{\rm kin} = \mathcal{H}_{\rm grav} \otimes \mathcal{H}_{\rm YM} \otimes \mathcal{H}_{\rm CS}^{\sigma_+} \otimes \mathcal{H}_{\rm CS}^{\sigma_-}$



Implementation gravity constraints:

- $F(A_{\sigma_\pm})$ picks up distributional contributions at punctures
- boundary DOF described by CS theory on S^2 with punctures $\mathcal{P}$

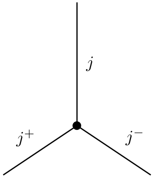
Hilbert space quantized SU(2) CS theory:

$$\mathcal{H}_{\rm CS}^{\sigma_\pm} \equiv \mathcal{H}_{k_\pm}^{\rm SU(2)}(\mathcal{P}, \{j_p^\pm\})$$

Quantum theory

Quantized BC imply coupling between j_p, j_p^+ and j_p^-:

- $\widehat{J}^i_\pm(p) := \frac{k_\pm}{4\pi} \lim_{\epsilon\to 0} \int_{D_\epsilon(p)} \widehat{F}(A_{\sigma_\pm})^i$ (limits exist strongly)
- $\widehat{J}^i(p) := \frac{2}{\kappa\beta} \lim_{\epsilon\to 0} \widehat{E}^i(D_\epsilon(p))$
- $D_\epsilon(p)$: disk of radius ϵ about $p \in \mathcal{P}$



quantized gravity BCs

$$\widehat{J}^i_\pm(p) = \pm \frac{\frac{a_H}{2\pi} d + \sigma_\pm^2}{\sigma_-^2 - \sigma_+^2} \widehat{J}^i(p) \quad \forall p \in \mathcal{P}$$

- $d = 2\left(\Psi_2 - 2\|\phi_1\|^2\right) + c$ (distortion parameter)

Solution BCs (as in matter-free case)

$$j_p = j_p^+ + j_p^-$$

Quantum theory

d, Ψ_2, c and ϕ_1 have to be interpreted as local operators

distortion operator

$$\widehat{d}(p) = 2\left(\widehat{\Psi}_2(p) - 2\|\widehat{\phi}_1\|^2(p)\right) + \widehat{c}(p)$$

Spherically symmetric configurations:

- defined as eigenvalues of distortion operator s.t. $\widehat{d}(p) = -\frac{2\pi}{a_H}$
- $\Rightarrow j_p^+ = j_p$ and $j_p^- = 0$ vice versa (spherically symmetric states)

Quantum theory

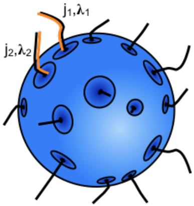
Implementation of matter BCs:

- $\Rightarrow \phi_1^I \to \widehat{\phi}_1^I(p)$
- $\Rightarrow \underleftarrow{\mathbf{E}}^I = -2\phi_1^I \mathrm{vol}_{S^2}$ defining equation for $\widehat{\phi}_e(p) := \|\widehat{\phi}_1\|(p)$

New BC in QT:

$$\widehat{\phi}_e(p) = 2\pi \frac{\widehat{Q}(p)}{\widehat{a}(p)} \qquad \text{(el.) charge-density operator}$$

- analogous for nontrivial magnetic charge



Subsection 2

Entropy

Entropy of the charged BH

1) Count gravity states only

- Fix the total charge Q_H (or Q'_H) and area a_H of the BH
- Count #(surface) CS-states s.t. exist state in $\mathcal{H}_{\mathrm{phys}}$ with

$$\langle \widehat{Q}_H \rangle = Q_H \quad \text{and} \quad \langle \widehat{a}_H \rangle = a_H$$

For distorted BHs: ϕ_1 arbitrary $\Rightarrow \widehat{\phi}_e$ not restricted. It follows that

$$S_{\mathrm{BH}} = \frac{\beta_k}{\beta} \frac{a_H}{4} + O(\ln a_H)$$

as before, independent of Q_H (or Q'_H) .

Inclusion of matter DOF

Can we include matter in the counting?

- No matter boundary term in symplectic structure, so no surface Hilbert space (?)
- Can still count field configurations $(\lambda_1, \lambda_2 \ldots)$ on the surface

Now details depend on choice of Q_H, Q'_H to be kept fixed:

- $\widehat{Q}_H$ behaves like $\sum_p \sqrt{\text{Casimir}}$ on λ_p. So finitely many λ-configurations for fixed Q_H.
- $\widehat{Q}'_H$ behave like $\sqrt{\text{Casimir}}$ in $\otimes_p \lambda_p$, so in general infinitely many λ-configurations for fixed Q'_H. For example

$$G = \mathrm{U}(1): \quad \widehat{Q}'_H \, |\underline{n}\rangle = g \sum_p n_p \, |\underline{n}\rangle \,, \qquad n_p \in \mathbb{Z}$$

$$G = \mathrm{SU}(2): \quad \widehat{Q}'_H = |J_{\text{tot}}| \text{ on } \otimes_p \, j_p = (j_1 + j_2 + \ldots) \oplus \ldots$$

Inclusion of matter DOF

2) Count gravity and matter states for U(1) keeping Q'_H fixed.

Regulator

$$|n_p| \leq N_{\text{max}} \qquad \text{for all } p$$

Then

$$N_{\text{YM}}(Q'_H, N_P) := \left|\{n_1, \ldots, n_{N_P} : |n_p| \leq N_{\text{max}}, \wedge g \sum_p n_p = Q'_H\}\right|$$
$$\approx (2N_{\max})^{N_P - 1}$$

under the assumption $N_{\text{max}} \gg g^{-1}|Q'_H|$.

Inclusion of matter DOF

Then

State counting

$$\mathcal{N}(a_H, Q_H) = \sum_{n=0}^{\infty} \sum_{d_{p_1}^{\pm}, \ldots, d_{p_n}^{\pm}=1}^{k+1} \delta(a_{\{d_i\}} - a_H) N_k(\{j_p^+\}) N_k(\{j_p^-\}) (2N_{\max})^{n-1}$$

- $a_{\{d_i\}} = 4\pi\beta \sum_{i=1}^{n} \sqrt{(d_{p_i}^+ + d_{p_i}^-)^2 - 1}, \; d_p = 2j_p + 1$
- $N_{k_\pm}(\{j_p^\pm\}) = \dim \mathcal{H}_k^{\mathrm{SU}(2)}(\mathcal{P}, \{j_p^\pm\}) \quad (k := k_+)$

Verlinde formula

$$N_k(\{j_i\}) = \int_0^{2\pi} \mathrm{d}\theta \, \frac{1}{\pi} \sin^2\left(\frac{\theta}{2}\right) \frac{\sin\left((2r+1)\frac{(k+2)\theta}{2}\right)}{\sin\left(\frac{(k+2)\theta}{2}\right)} \prod_{i=1}^{n} \frac{\sin\left(d_i \frac{\theta}{2}\right)}{\sin \frac{\theta}{2}}$$

- $r := \lfloor \frac{1}{k} \sum_{i=1}^{n} j_i \rfloor$

Inclusion of matter DOF

This gives

Entropy

$$S = \ln \mathcal{N} = \frac{\beta_k^{N_{\rm max}}}{\beta} \frac{a_H}{4 l_P^2} + \dots$$
$$\beta_k^{N_{\rm max}} \approx \frac{\ln 2}{\pi\sqrt{3}} + \frac{\ln(N_{\rm max})}{\pi\sqrt{3}}$$

- very similar to [Bombelli+Koul+Lee+Sorkin'86, Srednicki'93]
- $\beta_k^{N_{\rm max}}$ independent of CS k (compare [Ghosh, Noui, Perez '13])

Inclusion of matter DOF

3) Count gravity and matter states for U(1) keeping Q_H fixed. Asymptotics hard to determine, WIP [KE+HS+Selisko]. From simplified model expect

$$S = \frac{\beta'_k}{\beta}\frac{a_H}{4 l_P^2} + \left(\frac{Q_H}{g} - \frac{5}{2}\right)\ln a_H + \dots$$

- Similar to (but by no means same as) [Sen '12]

Section 4

The spherically symmetric limit

BH entropy

Spherically symmetric BHs in $\mathrm{SU}(2)$ approach: defined as eigenstates of distortion operator[Engle, Noui, Perez, Pranzetti '10] [Perez, Pranzetti '10]

$$\widehat{d}(p)\psi = -\frac{2\pi}{a_H}\psi \quad \forall p \in \mathcal{P}$$

$\Rightarrow$ for YM matter (requiring consistency with $\mathrm{SU}(2)$ framework):

$$\widehat{\phi}_e(p)\psi = 2\pi \tfrac{Q_H}{a_H}\psi$$

- direct coupling charge $\leftrightarrow$ area eigenvalues
- **But**: Both spectra discrete $\Rightarrow$ generically no solutions

BH entropy

Possible resolutions:

1. for $G = \mathrm{U}(1)$: Bohr compactification $\mathbb{R}_{\mathrm{Bohr}} \Rightarrow$ continuous charge eigenvalues
2. analytic continuation to $\beta = i$ [Achour+Noui+Perez'15,Achour+Mouchet+Noui'16]

Bohr compactification:

- for U(1) $\Rightarrow S_{\mathrm{BH}} = \frac{a_H}{4} + \dots$
- problem: only available for G Abelian

BH entropy

Analytic continuation to $\beta = i$

- continuous area eigenvalues ($\Rightarrow$ works for any YM theory) from

$$j_p \to \frac{1}{2}(-1 + i s_p), \qquad s_p \in \mathbb{R}$$

- CS level k becomes complex $\Rightarrow$ requires analytic continuation of Verlinde's formula

Dimension formula [Achour+Noui+Perez'15,A+Mouchet+N'16]

$$\dim \mathcal{H}^{\mathrm{CS}}_{k\to\infty}(\{s_p\}) \approx \frac{2}{\pi}\frac{1}{s\sqrt[3]{n}}\left(\frac{se}{2}\right)^n e^{\pi n s + i(1-n)\frac{\pi}{2}}$$

s_p: spin-network labels, $s := (\sum_{i=1}^n s_{p_i})/n$

BH entropy

For $Q_H \neq 0$: $s_p \approx Q(\lambda_p)\frac{a_H}{4\pi Q_H}$ (s_p fixed by λ_p), hence

$$s = \frac{\sum_{p=1}^{n} s_p}{n} = \frac{a_H}{4\pi Q_H}\frac{Q_H}{n} = \frac{a_H}{4\pi n}$$

- $\Rightarrow$ in highest order: $S_{\mathrm{BH}} = \frac{a_H}{4} + \ldots$ for any G
- charge discrete $\Rightarrow$ counting CS DOF $\leftrightarrow$ counting matter DOF
- $\Rightarrow$ charge contributes to lower order corrections in entropy:

BH entropy for $G = \mathrm{U}(1)$

$$S_{\mathrm{BH}} = \frac{a_H}{4} + \left(g^{-1} Q_H - 1\right) \ln a_H + \ldots$$

- Again: compare to [Sen '13]

Section 5

Summary & Outlook

Summary

- studied IHs in the presence of YM fields
- derived classical and quantum BC for (distorted) charged IHs
- computed entropy of (distorted) charged IHs with/without consideration of matter

We found: LQG picture works fine including quantized YM fields!
counting gravitational states only: For any G

$$S_{\mathrm{BH}} = \frac{\beta_k}{\beta}\frac{a_H}{4} + O(\ln a_H)$$

Including matter in the counting (U(1) case):

- Ensemble defined by fixed Q_H: Charge enters subleading order
- Ensemble defined by fixed Q'_H: Regulator! Modifies leading order, charge dependent subleading order.

Qualitative agreement with other approaches.

Summary

Problems with **spherically symmetric limit**:

- Requires constant charge density. Too few states for general G.
- Analytic continuation following Achour et al:

$$S_{\mathrm{BH}} = \frac{a_H}{4} + \left(g^{-1} Q_H - 1\right) \ln a_H + \dots$$

Have an idea how to include **magnetic charges**:

- Replace holonomies by exponentiated magnetic fluxes in HF algebra
- Introduce nonzero magnetic flux through variant KS representation

Outlook

What else can be done?

Technical stuff:

- entropy: leading and subleading term for all cases
- matter other than gauge fields
- cutoff: for field strength rather than for flux?
- other possibilities to implement spherical symmetry

Conceptual stuff:

- role of the regulator?
- can matter cancel the Immirzi parameter dependence?

Section 6

Appendix: magnetic charges

Magnetic charges

Turns out that YM BH stable requires $P_H \neq 0 \Rightarrow$ include magnetic charges (here: $G = \mathrm{U}(1)$)

- Stokes' theorem: exponentiated magnetic fluxes through surface S $\leftrightarrow$ holonomy along ∂S
- Hence: Consider algebra of electric fluxes $\mathbf{E}(S)$ and (exponentiated) magnetic fluxes $\mathbf{H}_S$

$$[\mathbf{E}(S), \mathbf{H}_{S'}] = g\, I(S, \partial S')\mathbf{H}_{S'}$$

- Choose some variant of Koslowski-Sahlmann rep. [K+S '12]

Representation

$$\widehat{\mathbf{H}}(S) = e^{ig\int_S \mathbf{B}^{(0)}} \mathbf{h}_{\partial S}, \quad \widehat{\mathbf{E}}(S)$$

- $\mathbf{B}^{(0)}$: background magnetic field

Magnetic charges

- states $|\gamma, \underline{n}|\gamma_0, \underline{m}\rangle$, γ_0: background closed graph carrying magnetic flux m
- gauge invariance $\Rightarrow$ no magnetic charge (for $\partial S = \emptyset$)
- $\Rightarrow$ introduce strings going from BH horizon to infinity

$$f^{(\text{string})}_{\sigma,\underline{n}^s}[\mathbf{A}] = \prod_{k=1}^{P} \mathbf{h}^{n_k}_{s_k}[\mathbf{A}],$$

- get new states $|\gamma, \underline{n}|\gamma_0, \underline{m}, \sigma, \underline{m}_0\rangle$
- $\Rightarrow$ **nontrivial magnetic charge**
- entropy computations may be generalized to this construction