Clock dependence and unitarity in quantum cosmology

Lucía Menéndez-Pidal

University of Nottingham

Work in collaboration with Steffen Gielen
arXiv: 2005.05357 + 2109.02660

22nd March 2022

Table of Contents

What is the problem of time (POT)?

Diffeomorphism invariance $\implies$ GR is a constrained system:

$$\mathcal{S}_{EH} = \int \mathrm{d}t\mathrm{d}^3x \left[p^{ab}\dot{h}_{ab} - N\mathcal{H}_{\perp}^{g} - N^a\mathcal{H}_a^g\right]$$

N and N^a (lapse and shift functions) are Lagrange multipliers. Hence,

$$\mathcal{H}_{\perp}^{g} = 0, \quad \mathcal{H}_a^g = 0$$

When we quantise we find

$$\hat{\mathcal{H}}_{\perp}^{g}\Psi = 0, \quad \hat{\mathcal{H}}_a^g\Psi = 0$$

How do we make observables evolve?

University of Nottingham UK | CHINA | MALAYSIA

Possible approaches to the POT

Choose time *before* quantisation

Reduced quantisation

- Solve the constraint classically and quantise only the true degrees of freedom
- Ambiguities in gauge fixing result in different quantum theories[1]

Choose time *after* quantisation

We start from a WdW equation[1] and then:

A Choose a clock[2] first and then an inner product

B Choose a too big Hilbert space and reduce it[3] (Dirac quantisation)

All these approaches are theory independent

[1]Ordering and other issues
[2]Why one and not another?
[3]Steps not straight forward, e.g., group averaging

Our take on the POT

1. Choose a cosmological model ($\Leftarrow$ easier metric)
2. Quantise using the Wheeler–DeWitt equation
3. Choose several dynamical variables as clock
4. Compare the resulting theories
5. Compare with the Dirac quantisation
6. Study singularity resolution

Table of Contents

Classical solutions

Ingredients

- Flat FLRW: $\mathrm{d}s^2 = -N(\tau)^2\mathrm{d}\tau^2 + a(\tau)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$
- Free massless scalar field ϕ
- A cosmological constant Λ from unimodular gravity

Unimodular gravity= GR+fixed determinant. Additional fields T^a can be introduced to recover full diffeomorphism invariance.

$$\mathcal{S}_{PUM} = \int \mathrm{d}^4x \left\{ \frac{\sqrt{-g}}{2\kappa}[R - 2\Lambda] + \Lambda\partial_a T^a - \frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi \right\}$$

$$= V_0 \int_{\mathbb{R}} \mathrm{d}\tau \left\{ \frac{3\dot{a}^2 a}{N\kappa} - Na^3\frac{\Lambda}{\kappa} + \Lambda\dot{T} + \frac{a^3}{2N}\dot{\phi}^2 \right\}$$

Classical solutions continued

Change of variables:

$$v = 2\sqrt{\frac{V_0}{3}}a^3,\ \pi_v = \sqrt{\frac{1}{12V_0}}\frac{\pi_a}{a^2},\ \lambda = V_0\Lambda,\ t = \frac{T}{V_0},$$
$$\varphi = 2\sqrt{\frac{3}{8}}\phi,\ \pi_\varphi = \frac{1}{2}\sqrt{\frac{8}{3}}\pi_\phi$$

where $\{t, \lambda\} = 1$. The cosmological constant is a constant of motion.

The action can be brought into Hamiltonian form with:

$$\mathcal{H} = \tilde{N}\left[-\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda\right] \overset{\text{constraint}}{\Longrightarrow} \mathcal{C} = -\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda = 0$$

Choice: $\tilde{N} = 1 \implies \frac{\mathrm{d}t}{\mathrm{d}\tau} = 1$. v volume

University of Nottingham UK | CHINA | MALAYSIA

The three different clocks

For $\lambda > 0$ the classical solutions are

$$v(t) = \sqrt{-\frac{\pi_\varphi^2}{\lambda} + 4\lambda(t - t_0)^2}, \qquad \varphi(t) = \frac{1}{2}\log\left|\frac{\pi_\varphi - 2\lambda(t - t_0)}{\pi_\varphi + 2\lambda(t - t_0)}\right| + \varphi_0$$

$$v(\varphi) = \frac{|\pi_\varphi|}{\sqrt{\lambda}|\sinh(\varphi - \varphi_0)|}, \qquad t(\varphi) = -\frac{\pi_\varphi}{2\lambda}\coth(\varphi - \varphi_0) + t_0$$

$$t(v) = t_0 - \mathrm{sgn}(\pi_v)\frac{1}{2}\sqrt{\frac{v^2}{\lambda} + \frac{\pi_\varphi^2}{\lambda^2}}, \quad \varphi(v) = \varphi_0 + \log\left|\frac{\pi}{\sqrt{\lambda}v} + \sqrt{\frac{\pi_\varphi^2}{\lambda v^2} + 1}\right|$$

The big bang/big crunch singularity is at $t_{\mathrm{sing}} = \frac{|\pi_\varphi|}{2\lambda}$, $\log\frac{v_{\mathrm{sing}}}{v_0} = -\infty$, $\varphi_{\mathrm{sing}} = \pm\infty$
Spatial infinity is at $t_\infty = \pm\infty$, $\log\frac{v_\infty}{v_0} = \infty$, $\varphi_\infty = \varphi_0$

Slow clocks vs fast clocks

Definition (Slow (fast) clocks)

Let $x \in \mathbb{R}$ be a dynamical variable used to express the variation of the remaining parameters of the universe. x is said to be *slow (or fast)* at the singularity/spatial infinity if the singularity/spatial infinity is reached at *a finite value* x_0 *(or at* $\pm\infty$*)*

In other words: If a clock is able to "push" the singularity/spatial infinity to the boundaries of its domain it is fast

- t-clock: slow at the singularity and fast at spatial infinity
- φ-clock: fast at the singularity and slow at spatial infinity
- v-clock: fast everywhere

Trajectories of $v(t)$ and $v(\varphi)$

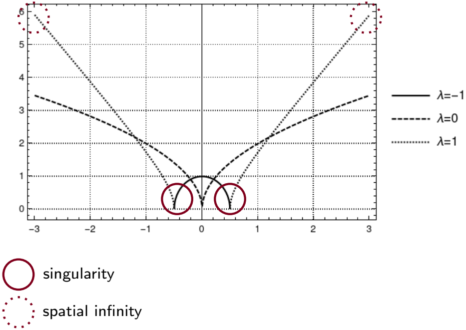

Figure: Classical trajectories of $v(t)$

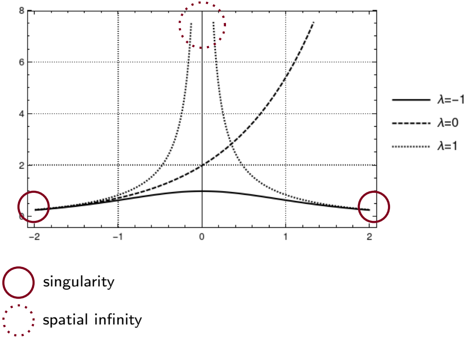

Figure: Classical trajectories of $v(\varphi)$

Why does the clock speed matter?

Gotay and Demaret (1983)

Conjecture:

- Classically: slow clock $\implies$ dynamics are truncated
- When quantising and demanding unitarity: state norm must be well defined everywhere in clock space $\implies$ artificially extending the solution to the whole domain of the clock $\implies$ singularity (or spatial infinity) resolution

We are going to verify this conjecture in our work

Similar studies

- Pawłowski and Ashtekar (2012) arXiv: 1011.3022 $\rightarrow$ Similar model with a fixed cosmological constant form a φ-clock perspective
- Gryb and Thébault (2019) arXiv: 1801.05789 and 1801.05826 $\rightarrow$ Same model from a t-clock perspective
- Gielen and Turok (2016) arXiv: 1510.00699 $\rightarrow$ Same model with a v-clock perspective
- Bojowald, Brizuela, Hernandez, Koop and Morales-Tecolt (2011) arXiv: 1011.3022 $\rightarrow$ Similar model with fixed cosmological constant using momenta

Table of Contents

Wheeler–DeWitt equation

$$\mathcal{C} = -\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda = 0 \implies g^{AB}\pi_A\pi_B + \lambda = 0, \quad g^{AB} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{v^2} \end{pmatrix}$$

Ordering problem: how do we go from here to a partial differential equation?
Answer: Replace $g^{AB}\pi_A\pi_B$ (which is flat) by the Laplace Beltrami operator $-\hbar^2\Box$

$$\hat{\mathcal{C}}\Psi = 0 \implies \left(\hbar^2\frac{\partial^2}{\partial v^2} + \frac{\hbar^2}{v}\frac{\partial}{\partial v} - \frac{\hbar^2}{v^2}\frac{\partial^2}{\partial \varphi^2} - i\hbar\frac{\partial}{\partial t}\right)\Psi(v,\varphi,t) = 0 \quad \text{(WdW)}$$

POT: how to extract time evolution from this equation?
Answer: A and B (and more?)

Answer A: choose a clock

t, φ and v are good clocks classically, hence both can a priori be good clocks for the quantum theory

The t-clock theory

$$i\hbar\partial_t\Psi = -\underbrace{\left(\hbar^2\partial_v^2 + \frac{\hbar^2}{v}\partial_v - \frac{\hbar^2}{v^2}\partial_\varphi^2\right)}_{\hat{\mathcal{H}}}\Psi$$

Schr. eq. $\implies$ Schr. inner product

$$\langle\Psi|\Phi\rangle_t = \int \mathrm{d}\varphi\mathrm{d}v\; v\bar{\Psi}\Phi$$

The φ-clock theory

Multiply (WdW) by v^2

$$\hbar^2\partial_\varphi^2\Psi = \underbrace{\left(\hbar^2(v\partial_v)^2 - iv^2\hbar\partial_t\right)}_{\hat{\mathcal{G}}}\Psi$$

K.G. eq. $\implies$ K.G. inner product

$$\langle\Psi|\Phi\rangle_\varphi = i\int \mathrm{d}t\frac{\mathrm{d}v}{v}\left(\bar{\Psi}\partial_\varphi\Phi - \Phi\partial_\varphi\bar{\Psi}\right)$$

University of Nottingham UK | CHINA | MALAYSIA

Choose a clock (continued)

the v-clock theory

Multiply (WdW) by v^2

$$\hbar^2 (v\partial_v)^2 \Psi = \underbrace{\left(\hbar^2 \partial_\varphi^2 + i v^2 \hbar \partial_t\right)}_{\hat{\mathcal{F}}} \Psi$$

K.G. eq. $\implies$ K.G. inner product

$$\langle \Psi | \Phi \rangle_v = i \int \mathrm{d}t \mathrm{d}\varphi \; v \left(\bar{\Psi} \partial_v \Phi - \Phi \partial_v \bar{\Psi}\right)$$

University of Nottingham
UK | CHINA | MALAYSIA

Choose a clock (continued)

The solutions to (WdW) are Bessel functions

$$\Psi(v,\varphi,t) = \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}k}{2\pi} e^{ik\varphi} e^{i\lambda\frac{t}{\hbar}} \left[\alpha(k,\lambda) J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right) + \beta(k,\lambda) J_{-i|k|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right)\right]$$
$$+ \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}\kappa}{2\pi} e^{\kappa\varphi} e^{i\lambda\frac{t}{\hbar}} \left[\gamma(\kappa,\lambda) J_{|\kappa|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right) + \epsilon(\kappa,\lambda) J_{-|\kappa|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right)\right]$$

$\alpha(k,\lambda)$, $\beta(k,\lambda)$, $\gamma(\kappa,\lambda)$ and $\epsilon(\kappa,\lambda)$ need to be constrained for two main reasons

- Normalisation: $\langle\Psi|\Phi\rangle < \infty$
- Time independence: We want our theory to be unitary

Choose a clock (continued)

Normalisation:

t-clock	φ-clock*	*v*-clock*
$\gamma(\kappa,\lambda)=\epsilon(\kappa,\lambda)=0$	$\epsilon(\kappa,\lambda)=0$	$\gamma(\kappa,\lambda)=\epsilon(\kappa,\lambda)=0$

*Klein–Gordon inner products are not positive definite but can easily be changed as solutions are already separating positive and negarive norm contributions

Choose a clock (continued)

Time independence

$$\partial_{t} \langle\Psi|\Phi\rangle_{t} = 0 \iff \left\langle \hat{\mathcal{H}}\Psi \middle| \Phi \right\rangle_{(L^2,\ v\mathrm{d}v\mathrm{d}\varphi)} = \left\langle \Psi \middle| \hat{\mathcal{H}}\Phi \right\rangle_{(L^2,\ v\mathrm{d}v\mathrm{d}\varphi)}$$

$$\partial_{\varphi} \langle\Psi|\Phi\rangle_{\varphi} = 0 \iff \left\langle \hat{\mathcal{G}}\Psi \middle| \Phi \right\rangle_{(L^2,\ \frac{\mathrm{d}v}{v}\mathrm{d}t)} = \left\langle \Psi \middle| \hat{\mathcal{G}}\Phi \right\rangle_{(L^2,\ \frac{\mathrm{d}v}{v}\mathrm{d}t)}$$

$$\partial_{v} \langle\Psi|\Phi\rangle_{v} = 0 \iff \left\langle \hat{\mathcal{F}}\Psi \middle| \Phi \right\rangle_{(L^2,\ v\mathrm{d}t\mathrm{d}\varphi)} = \left\langle \Psi \middle| \hat{\mathcal{F}}\Phi \right\rangle_{(L^2,\ v\mathrm{d}t\mathrm{d}\varphi)}$$

In a nutshell,

the unitarity requirement is equivalent to a self-adjointness problem

Choose a clock (continued)

Symmetric operators fall in three categories:

- Self-adjoint operators (like $\hat{\mathcal{F}}$)
- Are not self-adjoint but admit self-adjoint extensions (like $\hat{\mathcal{G}}$ or $\hat{\mathcal{H}}$)
- Are not self-adjoint and do not admit self-adjoint extensions

Because $\hat{\mathcal{F}}$ is already self-adjoint, we are done for the v-clock theory

Choose a clock (continued)

$\hat{\mathcal{H}}$ and $\hat{\mathcal{G}}$ are not self-adjoint $\Longrightarrow$ boundary conditions have to be imposed

t-clock theory

$$\int \mathrm{d}\varphi \left[v\bar{\Psi}\partial_v\Phi - v\Phi\partial_v\bar{\Psi}\right]_{v=0} = 0$$

- Real combinations of Bessel functions

φ-clock theory

$$\int \mathrm{d}t \left[v\bar{\Psi}\partial_v\Phi - v\Phi\partial_v\bar{\Psi}\right]^{v=\infty} = 0$$

- Real combination of Bessel functions

$$\Psi_t \sim \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}k}{2\pi}\, e^{ik\varphi}e^{i\lambda\frac{t}{\hbar}}\alpha(k,\lambda)\,\mathrm{Re}\left[e^{i\vartheta(k)-i|k|\log\sqrt{\frac{\lambda}{\lambda_0}}}J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right)\right]$$

$$\Psi_\varphi \sim \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}k}{2\pi} e^{ik\varphi}e^{i\lambda\frac{t}{\hbar}}\alpha(k,\lambda)\,\mathrm{Re}\left[\sqrt{\frac{\sinh\left((|k|-i\kappa_0(\lambda))\frac{\pi}{2}\right)}{\sinh\left((|k|+i\kappa_0(\lambda))\frac{\pi}{2}\right)}}J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right)\right]$$

University of Nottingham UK | CHINA | MALAYSIA

Choose a clock (conclusion)

Remark:

- In the t-theory, the boundary condition is at $v = 0$ (classical singuarity), where classically the clock is slow
- In the φ-theory, the boundary condition is at $v = \infty$ (spatial infinity), where again classically the clock is slow

In conclusion:

1. Unitarity implies a boundary condition at the slow point of the clock
2. Different clock choices lead to different boundary conditions

Is this a consequence of choosing a clock first and then building an inner product?

What about Dirac quantisation?

Answer B: Dirac quantisation

We have two WdW equations

$$\hat{\mathcal{C}}_1\Psi = 0, \implies \left(\hbar^2\frac{\partial^2}{\partial v^2} + \frac{\hbar^2}{v}\frac{\partial}{\partial v} - \frac{\hbar^2}{v^2}\frac{\partial^2}{\partial\varphi^2} - i\hbar\frac{\partial}{\partial t}\right)\Psi = 0 \quad \text{(WdW1)}$$

$$\hat{\mathcal{C}}_2\Psi = 0, \implies \left(\hbar^2\left(v\frac{\partial}{\partial v}\right)^2 - \hbar^2\frac{\partial^2}{\partial\varphi^2} - i\hbar v^2\frac{\partial}{\partial t}\right)\Psi = 0 \quad \text{(WdW2)}$$

Note that (WdW2)$=v^2$(WdW1) The interpretation of the second order derivatives as a $\hbar^2\Box$ for different metrics motivates different kinematical inner products

$$\langle\Psi|\Phi\rangle_{\text{kin}_1} = \int \mathrm{d}t\mathrm{d}\varphi\mathrm{d}v \; v\bar{\Psi}\Phi \quad \text{(IP1)}$$

$$\langle\Psi|\Phi\rangle_{\text{kin}_2} = \int \mathrm{d}t\mathrm{d}\varphi\frac{\mathrm{d}v}{v} \; \bar{\Psi}\Phi \quad \text{(IP2)}$$

University of Nottingham
UK | CHINA | MALAYSIA

Dirac quantisation (continued)

Demanding $\hat{\mathcal{C}}_1$ and $\hat{\mathcal{C}}_2$ to be self-adjoint with respect to the inner products (IP1) and (IP2) corresponds to self-adjointness of the one dimensional operators

$$\hat{\mathcal{D}}_1 = \hbar^2 \left(-\frac{\partial^2}{\partial v^2} - \frac{k^2 + \frac{1}{4}}{v^2} \right), \quad \hat{\mathcal{D}}_2 = -\hbar^2 \left(v \frac{\partial}{\partial v} \right)^2 - \lambda v^2$$

Recall,

$$\hat{\mathcal{H}} = \left(\hbar^2 \frac{\partial^2}{\partial v^2} + \frac{\hbar^2}{v} \frac{\partial}{\partial v} - \frac{\hbar^2}{v^2} \frac{\partial^2}{\partial \varphi^2} \right), \quad \hat{\mathcal{G}} = \left(\hbar^2 \left(v \frac{\partial}{\partial v} \right)^2 - i v^2 \hbar \frac{\partial}{\partial t} \right)$$

Despite being two dimensional, study $\hat{\mathcal{H}}$ and $\hat{\mathcal{G}}$ reduces to analysing $\hat{\mathcal{D}}_1$ and $\hat{\mathcal{D}}_2$

Dirac quantisation (conclusion)

The self-adjointness problem remains in the Dirac quantisation

We multiplied the Wheeler-DeWitt equation by a phase function to find different theories, but $\hat{\mathcal{C}}\Psi = 0$ and $\hat{\mathcal{N}}\hat{\mathcal{C}}\Psi = 0$ have the same solutions, this is consequence of reparametrisation invariance

How do we choose between the different Wheeler–DeWitt equations?

Dirac quantisation (parenthesis)

P. A. Höhn, A. R. H. Smith and M. P. Lock 2019 and 2021
arXiv:1912.00033 and 2007.00580
$\rightarrow$ framework for clock changes

Our work *does not* contradict theirs as the framework is different: they work with clock changes given a WdW equation and instead we develop a theory for two different WdW equations

Table of Contents

Dynamics of the v-theory

General solution:

$$\Psi(v,\varphi,t)_{sc} = \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}k}{2\pi} e^{ik\varphi} e^{i\lambda\frac{t}{\hbar}} \left[\alpha(k,\lambda) J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar} v\right) + \beta(k,\lambda) J_{-i|k|}\left(\frac{\sqrt{\lambda}}{\hbar} v\right)\right]$$

- $\alpha(k,\lambda) = 0$ (only outgoing modes)
- $\beta(k,\lambda)$ sharply peaked around $k = k_c$ and $\lambda = \lambda_c$

Expanding around $v = 0$

- Classically: $t(v) = \frac{\hbar|k_c|}{2\lambda_c} + \frac{v^2}{4\hbar|k_c|} - \frac{\lambda_c v^4}{16\hbar^3|k_c|^3} + O(v^6)$
- Quantum theory: $\langle \Psi_{sc}(v)|\, t\, |\Psi_{sc}(v)\rangle = \langle t(v)\rangle_{\Psi_{sc}} = \frac{\hbar|k_c|}{2\lambda_c} + \frac{v^2}{4\hbar|k_c|} - \frac{\lambda_c v^4}{16\hbar^3(|k_c|+|k_c|^3)} + O(v^6)$

No strong divergence from the classical theory

Dynamics of the t-clock theory

Recall the general solution

$$\Psi_{sc} \sim \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}k}{2\pi}\; e^{ik\varphi} e^{i\lambda\frac{t}{\hbar}} \alpha(k,\lambda)\,\mathrm{Re}\left[e^{i\vartheta(k) - i|k|\log\sqrt{\frac{\lambda}{\lambda_0}}} J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar} v\right)\right]$$

- $\vartheta(k) = 0$ for simplicity
- $\alpha(k,\lambda)$ sharply peaked around $k = k_c$ and $\lambda = \lambda_c$
- expectation values $= \langle \Psi_{sc}(t)|\, v\, |\Psi_{sc}(t)\rangle = \langle v(t)\rangle_{\Psi_{sc}}$
- criteria for singularity resolution: $\langle v(t)\rangle_{\Psi_{sc}} > C > 0$

Dynamics for small v

Bessel function asymptote for small arguments

$$J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right) \longrightarrow \frac{e^{i|k|\log\left(\frac{\sqrt{\lambda}}{2\hbar}v\right)}}{\Gamma(1+i|k|)}$$

With the prefactor

$$e^{i\vartheta(k)-i|k|\log\sqrt{\frac{\lambda}{\lambda_0}}}$$

Ψ_{sc} looks like a superposition of plane waves independent of λ outgoing from and ingoing to the singularity

Singularity resolution

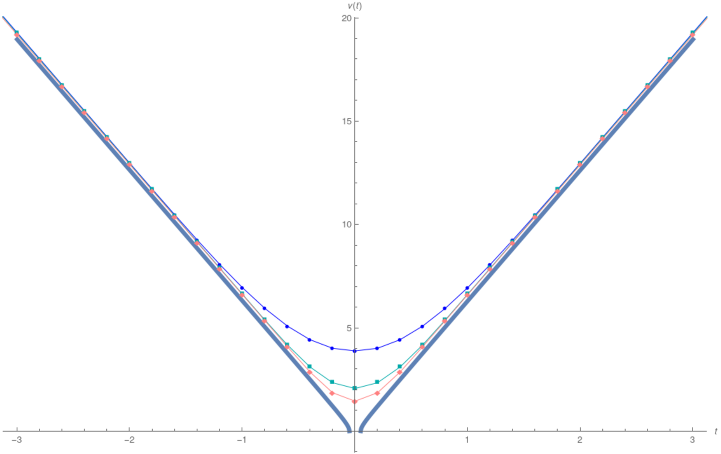

Figure: $\langle v(t) \rangle_{\Psi_{sc}}$ for different semiclassical states

Dynamics of the φ-clock theory

Recall the general solution

$$\Psi_{sc} \sim \int \frac{\mathrm{d}\lambda}{2\pi\hbar}\frac{\mathrm{d}k}{2\pi} e^{ik\varphi} e^{i\lambda\frac{t}{\hbar}} \alpha(k,\lambda)\,\mathrm{Re}\left[\sqrt{\frac{\sinh\left((|k| - i\kappa_0(\lambda))\frac{\pi}{2}\right)}{\sinh\left((|k| + i\kappa_0(\lambda))\frac{\pi}{2}\right)}} J_{i|k|}\left(\frac{\sqrt{\lambda}}{\hbar}v\right)\right]$$

- $\kappa_0(\lambda) = 0$ for simplicity
- $\alpha(k,\lambda)$ sharply peaked around $k = k_c$ and $\lambda = \lambda_c$
- expectation values $\langle\Psi_{sc}(\varphi)|\, v\, |\Psi_{sc}(\varphi)\rangle = \langle v(\varphi)\rangle_{\Psi_{sc}}$ and $\langle\Psi_{sc}(\varphi)|\, t\, |\Psi_{sc}(\varphi)\rangle = \langle t(\varphi)\rangle_{\Psi_{sc}}$
- criteria for quantum recollapse $\langle v(\varphi)\rangle_{\Psi_{sc}} < C' < \infty$

Quantum recollapse

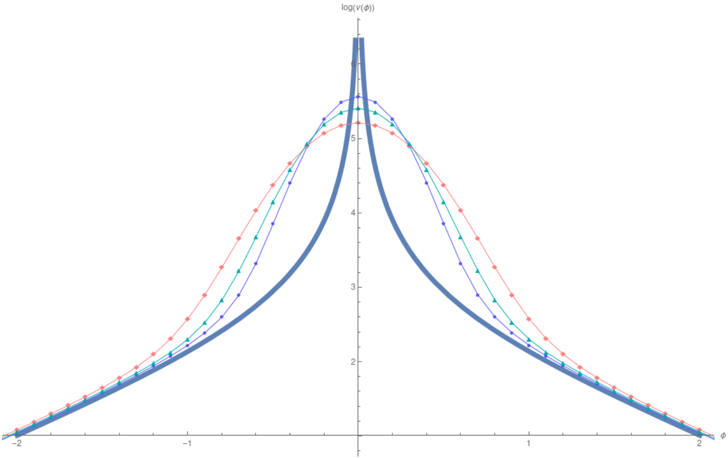

Figure: $\langle v(\varphi)\rangle_{\Psi_{sc}}$ for different semiclassical states

Quantum recollapse

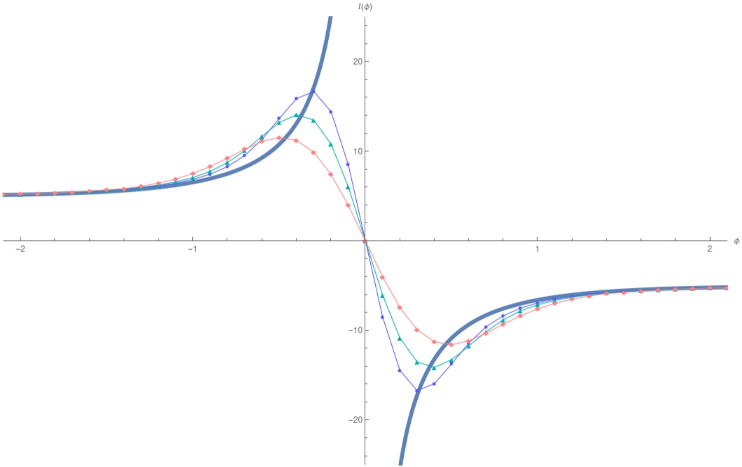

Figure: $\langle t(\varphi)\rangle_{\Psi_{sc}}$ for different semiclassical states

Quantum recollapse

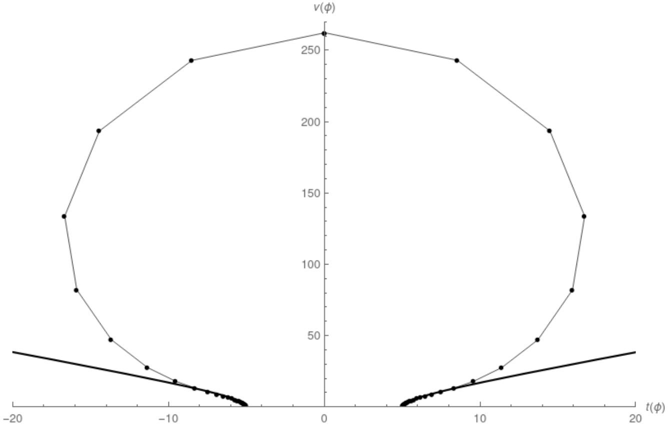

Figure: $\langle t(\varphi)\rangle_{\Psi_{sc}}$ versus $\langle v(\varphi)\rangle_{\Psi_{sc}}$ for the same state

Interpretation of the dynamics

- The v-clock shows no strong divergence from the classical theory
- The t-clock theory shows singularity resolution $(\langle v(t)\rangle_{\Psi_{sc}} > C > 0)$
- The φ-clock theory shows a quantum recollapse $(\langle v(\varphi)\rangle_{\Psi_{sc}} < C' < \infty)$
- The quantum behaviour can be interpreted as a reflection from the boundary (either $v = \infty$ or $v = 0$) $\implies$ The boundary condition "decides" whether the theory is singular or not
- Far from the boundary, the classical and quantum curves look very similar

Table of Contents

Recap

	Unitarity	Bound. cond.	Singularity res.	Infinity res.
v	✔	✘	✘	✘
t	✘	✔	✔	✘
φ	✘	✔	✘	✔

Table: Differences and similarities between the clocks

A few things to take from this talk

- The problem of time has many nuances
- Quantum cosmology is a good testing ground
- Unitarity requirements are what lead to different theories and,
- They are present in different quantisation schemes
- Singularity resolution is not a feature of the theory, it is a feature of the clock $\implies$ clock choices should be given more attention when studying these models

Thank you!

Contact: lucia.menendez-pidal@nottingham.ac.uk

University of Nottingham
UK | CHINA | MALAYSIA