Dirac Observables and (b, v)-Type Variables for Effective Polymer Black Holes

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based on

arXiv:1911.12646 [gr-qc] & arXiv:1912.xxxxx [gr-qc]

with

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Motivation

- BH are of interest in all main QG approaches
- May provide better understanding of full LQG
- Singularity resolution in AdS/CFT [Bodendorfer, Schäfer, Schliemann '16, Bodendorfer, FM, JM '18]

- Construct effective model inspired by LQG
- Lot of previous effort: [Ashtekar, Olmedo, Singh '18; Bianchi, Christodoulou, D'Ambrosio, Haggard, Rovelli '18; Olmedo, Saini, Singh '17; Perez '17; Corichi, Singh '16; Modesto '10; Pullin '07-,...]
- No consensus about BHs in LQG community

- BH interior has Kantowski-Sachs (cosmology) structure
- Use techniques of LQC (polymerisation)
- Subtleties: Dirac observables / Different schemes

Plan of the Talk

- PART I: Dirac observables (Johannes)
  - Classical theory
  - Effective quantum theory and previous models
- PART II: (b, v)-type variables (Fabio)
  - Adapted variables for effective polymer BHs
  - Overcome previous limitations for physical viability
- Conclusions and outlook

PART I

Mass and Horizon Dirac Observables in Polymer BH Models

Classical Setting

Spherically symmetric and static spacetime

[Vakili '18; Cavaglia, Alfaro, Filippov '94]

$$ds^2 = -\bar{a}(r)dt^2 + N(r)dr^2 + \mathscr{b}(r)^2 d\Omega_2^2$$

For $\bar{a}\,, N < 0$ t spacelike, r timelike → Kantowski-Sachs-Cosmology

Regularisation: L_o and $\mathscr{L}_o = \int_0^{L_o} \sqrt{\bar{a}}\Big|_{r=r_{\mathbf{ref}}} \mathrm{d}t$

$$\sqrt{a} = \int_0^{L_o} \sqrt{\bar{a}}\,\mathrm{d}t = L_o\sqrt{\bar{a}}\,, \quad n = Na$$

Connection variables for the interior

[Ashtekar, Olmedo, Singh '18; Olmedo, Saini, Singh '17; Corichi, Singh '16; Pullin '07-,...]

$$ds^2 = -N_T^2(T)\,\mathrm{d}T^2 + \frac{p_b^2(T)}{L_o^2|p_c(T)|}\,\mathrm{d}x^2 + |p_c(T)|\,\mathrm{d}\Omega_2^2$$

$$T = r \quad , \quad x = t \quad , \quad |p_c| = \mathscr{b}^2 \quad , \quad p_b^2 = -a\mathscr{b}^2 \quad , \quad -N = N_T^2$$

Classical Integration Constants

$$b(T) = \pm\gamma\sqrt{Ae^{-T} - 1} \quad , \quad c(T) = c_o e^{-2T}$$

$$p_b(T) \overset{\mathcal{H}\approx 0}{=} -\frac{2cp_c}{b + \frac{\gamma^2}{b}} = \mp\frac{2c_o p_c^o}{\gamma}\sqrt{\frac{e^T}{A}\left(1 - \frac{e^T}{A}\right)} \quad , \quad p_c(T) = p_c^o e^{2T} \ .$$

Solving EoM $\Rightarrow$ two integration constants c_o, p_c^o, $A = e^{T_o} = 1$ (w.l.o.g.)

Express $a = p_b^2/|p_c|$ in terms of $b = \sqrt{|p_c|}$

$$b = \sqrt{|p_c^o|}e^T \quad , \quad a(b) = \frac{4c_o^2|p_c^o|}{\gamma^2 L_o^2}\left(\frac{\sqrt{|p_c^o|}}{b} - 1\right) \quad \Rightarrow \quad R_{hor} = \sqrt{|p_c^o|}$$

Line element

Redefining coordinates $\tau = \sqrt{|p_c^o|}e^T$, $y = \frac{2c_o\sqrt{|p_c^o|}}{\gamma L_o}x$

$$\mathrm{d}s^2 = -\frac{1}{\frac{R_{hor}}{\tau} - 1}\mathrm{d}\tau^2 + \left(\frac{R_{hor}}{\tau} - 1\right)\mathrm{d}y^2 + \tau^2\mathrm{d}\Omega_2^2 \ ,$$

Dirac Observables

Canonical degrees of freedom

4 phase space d.o.f + 1 first class constraint ⇒ 2 Dirac observables

$$\mathcal{R}_{hor} = \sqrt{|p_c|}\left(\frac{b^2}{\gamma^2}+1\right) \overset{\text{on-shell}}{=} R_{hor} = \sqrt{|p_c^o|} \quad , \quad \mathcal{D} = c p_c \overset{\text{on-shell}}{=} c_o p_c^o$$

Only $\mathcal{R}_{hor}$ physical, $\mathcal{D}$ is irrelevant (not in metric + fiducial cell dep.)

Residual diffeomorphisms

Redefining coordinates $y = \frac{2c_o\sqrt{|p_c^o|}}{\gamma L_o}x$ changes $\bar{a} \mapsto \left(\frac{2c_o\sqrt{|p_c^o|}}{\gamma L_o}\right)^{-2}\bar{a}$

$$\sqrt{a} = L_o\sqrt{\bar{a}} = \int_0^{L_o} \mathrm{d}x\sqrt{\bar{a}} = \int_{y(0)}^{y(L_o)} \mathrm{d}y\sqrt{\tilde{\bar{a}}} = y(L_o)\sqrt{\tilde{\bar{a}}} = \sqrt{a}$$

Transformation not present on phase space!

Canonical Dirac observables do **not know** about this remaining freedom.
The physical metric **is affected**.

Conclusions Classical Theory

- Fiducial cell → residual diffeomorphisms
- Not present on the phase space → visible through L_o dependence
- go back to metric → $\mathcal{D}$ can be absorbed

- There are **two Dirac observables**, only one physically relevant
- T, x rescaling absorbs one Dirac observable
- Identification: Physical observable is $R_{hor} = 2M_{BH}$

Warning!!!

Observables of quantum theory in classical regime
≠ Observables of classical theory

There, the second Dirac observable might be **physically relevant**!

⟶ Examples

Define new variables

$$(p_b)^2 = -8v_2 \quad , \quad |p_c| = (24v_1)^{\frac{2}{3}} \ ,$$

$$b = \text{sign}(p_b)\,\frac{\gamma}{4}\,\sqrt{-8v_2}\;P_2 \quad , \quad c = -\text{sign}(p_c)\,\frac{\gamma}{8}\,(24v_1)^{\frac{1}{3}}\;P_1 \ .$$

Polymerisation with constant λ_i and $P_i \mapsto \sin(\lambda_i P_i)/\lambda_i$

$$\mathscr{b} = \left(\frac{3v_1}{2}\right)^{\frac{1}{3}} = \frac{\mathscr{L}_o}{\lambda_2}\left(3DC^2\lambda_1^2\right)^{\frac{1}{3}}\frac{\left(\frac{\lambda_2^6}{16C^2\lambda_1^2\mathscr{L}_o^6}\left(\frac{\mathscr{L}_o r}{\lambda_2}+\sqrt{1+\frac{\mathscr{L}_o^2 r^2}{\lambda_2^2}}\right)^6+1\right)^{\frac{1}{3}}}{\left(\frac{\mathscr{L}_o r}{\lambda_2}+\sqrt{1+\frac{\mathscr{L}_o^2 r^2}{\lambda_2^2}}\right)} \ ,$$

$$a = \frac{v_2}{2b^2} \ , \quad v_2 = 2\mathscr{L}_o^2\left(\frac{\lambda_2}{\mathscr{L}_o}\right)^2\left(1+\frac{\mathscr{L}_o^2 r^2}{\lambda_2^2}\right)\left|1-\frac{3CD}{2\lambda_2}\frac{1}{\sqrt{1+\frac{\mathscr{L}_o^2 r^2}{\lambda_2^2}}}\right|$$

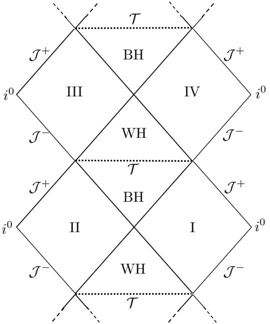
Again two integration constants C, D, now: can't absorb D in r

Mass Dirac Observables

$$F_Q(v_1, P_1, P_2) \overset{\text{on-shell}}{=} \left(\frac{3}{2}D\right)^{\frac{4}{3}} \frac{C}{\mathscr{L}_o}$$

$$\bar{F}_Q(v_1, P_1, P_2) \overset{\text{on-shell}}{=} \frac{3CD\mathscr{L}_o}{\lambda_2^2} \left(3DC^2\lambda_1^2\right)^{\frac{1}{3}}$$

Both Dirac observables have physical meaning!



Asymptotic Schwarzschild regions

For $r \to \pm\infty$ spacetime is asymptotically described by

$$ds_+^2 \simeq -\left(1 - \frac{F_Q}{b}\right) d\tau^2 + \frac{1}{1 - \frac{F_Q}{b}} db^2 + b^2 d\Omega_2^2$$

Similar for ds_-^2 with $\bar{F}_Q$.
Identify $2M_{BH} = F_Q$ and $2M_{WH} = \bar{F}_Q \Leftrightarrow$ Initial cond. $\mathcal{K}(b_i)$ and $R(b_i)$

Horizon Dirac Observables [Bodendorfer, FM, JM '19]

(generalised) μ_o-schemes

[Ashtekar, Olmedo, Singh '18; Corichi, Singh '16;...] and similar [Modesto '09; '10]

$$\dot{b} = -\frac{1}{2}\left(\frac{\sin(\delta_b b)}{\delta_b} + \frac{\gamma^2 \delta_b}{\sin(\delta_b b)}\right) \quad , \quad \dot{c} = -2\frac{\sin(\delta_c c)}{\delta_c} \, ,$$

$$\dot{p}_b = \frac{p_b}{2}\cos(\delta_b b)\left(1 - \frac{\gamma^2 \delta_b^2}{\sin(\delta_b b)^2}\right) \quad , \quad \dot{p}_c = 2 p_c \cos(\delta_c c) \, .$$

Horizon Dirac observables

$$\mathcal{R}_{BH} = \left[\frac{p_c \sin(\delta_c c)}{2}\left(\frac{\tan\left(\frac{\delta_c c}{2}\right)}{B_o^2}\left(\frac{b_o + \cos(\delta_b b)}{b_o - \cos(\delta_b b)}\right)^{\frac{2}{b_o}} + \frac{B_o^2}{\tan\left(\frac{\delta_c c}{2}\right)}\left(\frac{b_o - \cos(\delta_b b)}{b_o + \cos(\delta_b b)}\right)^{\frac{2}{b_o}}\right)\right]^{\frac{1}{2}} \, ,$$

$$\mathcal{R}_{WH} = \left[\frac{p_c \sin(\delta_c c)}{2}\left(B_o^2 \tan\left(\frac{\delta_c c}{2}\right)\left(\frac{b_o + \cos(\delta_b b)}{b_o - \cos(\delta_b b)}\right)^{\frac{2}{b_o}} + \frac{1}{B_o^2 \tan\left(\frac{\delta_c c}{2}\right)}\left(\frac{b_o - \cos(\delta_b b)}{b_o + \cos(\delta_b b)}\right)^{\frac{2}{b_o}}\right)\right]^{\frac{1}{2}} \, .$$

For [Modesto '09; '10] also mass observables can be constructed

Two d.o.f. ⇒ Revisit arguments of [Ashtekar, Olmedo, Singh '18; Corichi, Singh '16;...]

General Statement

General observation

Fiducial cell dependence of one poly. scale $\Longleftrightarrow$ Two physical Dirac observables

Both scales **fiducial cell independent** [Bodendorfer, FM, JM '19]
$\Rightarrow$ Only **one** Dirac observable

Fiducial cell dependence

- Two masses are unrelated
- More freedom
- Restriction of initial conditions $M_{WH} = f(M_{BH})$ by quantum conditions
- Possibility to circumvent?

Fiducial cell independence

- One observable encodes both masses
- A relation $M_{WH} = f(M_{BH})$ is selected by dynamics
- $M_{BH} \leftrightarrow M_{WH}$ symmetry $\Rightarrow M_{WH} \propto M_{BH}^{\pm 1}$
- We found a model for -1

PART II

(b,v)-Type Variables for Effective Polymer Black Holes

New Canonical Variables: Curvature Variables

Previous model

$$\frac{P_1(\mathscr{b})}{\mathscr{L}_o} = \left(\frac{2}{3D}\right)^{\frac{1}{3}} \frac{2M_{BH}}{\mathscr{b}^3} \quad \propto \quad \sqrt{\mathcal{K}} \qquad \text{iff} \qquad D \text{ mass-independent}$$

$$\downarrow$$

mass indep. curvature upper bound for specific relations $M_{WH}(M_{BH})$

New variables directly related to curvature

$$v_k = \left(\frac{3}{2}v_1\right)^{\frac{2}{3}} \frac{1}{P_2} \quad , \quad v_j = v_2 - \frac{3v_1P_1}{2P_2} \quad , \quad k = \left(\frac{3}{2}v_1\right)^{\frac{1}{3}} P_1P_2 \quad , \quad j = P_2$$

s.t.

$$k(\mathscr{b}) \approx R_{\mu\nu\alpha\beta}\epsilon^{\mu\nu}\epsilon^{\alpha\beta} = \frac{2M_{Misner-Sharp}(\mathscr{b})}{\mathscr{b}^3} \xrightarrow{\text{on-shell}} \frac{2M_{BH}}{\mathscr{b}^3} \propto \sqrt{\mathcal{K}}$$

physical viability independently of the relation between the masses

Similar to (b, v)-variables in LQC, where $R \propto b^2$.

Classical Theory in (v_k, k, v_j, j)-Variables

Hamiltonian

$$H_{cl} = \sqrt{n}\mathcal{H}_{cl} \qquad , \qquad \mathcal{H}_{cl} = 3v_k k j + v_j j^2 - 2 \approx 0$$

One fiducial cell indep. Dirac observable (for two integration constants)

$$F = k\left(v_k j\right)^{\frac{3}{2}} = (D)^{\frac{3}{2}}\, C$$

Express $a = \frac{v_j j + v_k k}{2 v_k j^2}$ in terms of $b = \sqrt{v_k j}$

$$b = \sqrt{D} r \quad , \quad a(b) = \frac{1}{D}\left(1 - \frac{F}{b}\right) \quad \Rightarrow \quad 2M_{BH} = F$$

Line element

Coordinate redefinition $\tau = t/\sqrt{D}$, $b = \sqrt{D} r$

$$ds^2 = -\left(1 - \frac{2M_{BH}}{b}\right) d\tau^2 + \frac{1}{1 - \frac{2M_{BH}}{b}} db^2 + b^2 d\Omega_2^2$$

Polymerisation of the Model

- Interior: r time-like → homog. Cauchy slices with topology $\mathbb{R} \times \mathbb{S}^2$
- On-shell interpretation:

$$k(\mathscr{b}) = \frac{2M_{BH}}{\mathscr{b}^3} \propto \sqrt{\mathcal{K}} \qquad , \qquad j(\mathscr{b})\mathscr{L}_o = \frac{1}{\mathscr{b}}\left(\frac{3D}{2}\right)^{\frac{1}{3}}$$

- Polymerisation (constant $\lambda \to \mu_o$-scheme):

$$k \mapsto \frac{\sin(\lambda_k k)}{\lambda_k} \qquad , \qquad j \mapsto \frac{\sin(\lambda_j j)}{\lambda_j}$$

  - $[\lambda_k] = L^2$ inverse curvature scale (**large curv. qu.-effects**)
  - $[\lambda_j/\mathscr{L}_o] = L$ length scale (**small areal radius qu.-effects**)

- Effective Hamiltonian

$$H_{\text{eff}} = \sqrt{n}\mathcal{H}_{\text{eff}} \,, \quad \mathcal{H}_{\text{eff}} = 3v_k \frac{\sin\left(\lambda_k k\right)}{\lambda_k} \frac{\sin\left(\lambda_j j\right)}{\lambda_j} + v_j \frac{\sin^2\left(\lambda_j j\right)}{\lambda_j^2} - 2 \approx 0$$

Solutions of the Effective Dynamics

Metric coefficients (with $x = \mathscr{L}_o r/\lambda_j$):

$$b^2(x) = \frac{1}{2}\left(\frac{\lambda_k}{M_{BH}M_{WH}}\right)^{\frac{2}{3}} \frac{1}{\sqrt{1+x^2}} \frac{M_{BH}^2\left(x+\sqrt{1+x^2}\right)^6 + M_{WH}^2}{\left(x+\sqrt{1+x^2}\right)^3}$$

$$\frac{a(x)}{\lambda_j^2} = 2\left(\frac{M_{BH}M_{WH}}{\lambda_k}\right)^{\frac{2}{3}}\left(1-\left(\frac{M_{BH}M_{WH}}{\lambda_k}\right)^{\frac{1}{3}}\frac{1}{\sqrt{1+x^2}}\right)\frac{\left(1+x^2\right)^{\frac{3}{2}}\left(x+\sqrt{1+x^2}\right)^3}{M_{BH}^2\left(x+\sqrt{1+x^2}\right)^6+M_{WH}^2}$$

Main features

- solution extends to exterior, $r \in (-\infty, \infty)$, two asymp. regions
- two horizons at $r = r_s^{(\pm)}$ s.t. $a(r) = 0$:

$$b_\pm \simeq 2M_{BH/WH} + \text{quant. corrections } (\to 0 \text{ as } \lambda \to 0)$$

- b has non-zero minimum (*transition surface*)
- Two integration constants M_{BH}, M_{WH}, both physically relevant
- λ_j does not appear in the final metric

Integration Constants

Dirac observables

$$F_Q = \frac{\sin(\lambda_k k)}{\lambda_k} \cos\left(\frac{\lambda_k k}{2}\right) \left(\frac{2 v_k}{\lambda_j \cot\left(\frac{\lambda_j j}{2}\right)} \right)^{\frac{3}{2}} ,$$

$$\bar{F}_Q = \frac{\sin(\lambda_k k)}{\lambda_k} \sin\left(\frac{\lambda_k k}{2}\right) \left(\frac{2 v_k}{\lambda_j} \cot\left(\frac{\lambda_j j}{2}\right) \right)^{\frac{3}{2}}$$

Asymptotic Schwarzschild regions

For $r \to \pm\infty$ spacetime is asymptotically described by

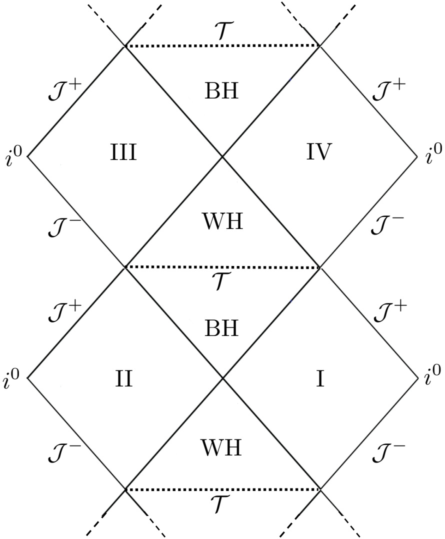
$$ds_+^2 \simeq -\left(1 - \frac{F_Q}{b}\right) d\tau^2 + \left(1 - \frac{F_Q}{b}\right)^{-1} db^2 + b^2 d\Omega_2^2$$

Similarly for ds_-^2 with $\bar{F}_Q$.

$\Longrightarrow$ Identify $2M_{BH} = F_Q$ and $2M_{WH} = \bar{F}_Q$

Note: M_{BH} and M_{WH} **independent!**

Quantum-Corrected Effective Spacetime Structure



- Eff. metric smooth in the whole r-domain $r \in (-\infty, +\infty)$
- Quantum effects relevant in large curvature regime
- Singularity resolved by transition surface $\mathcal{T}$

  (BH-to-WH transition)
- Infinetely many trapped (BH) and anti-trapped (WH) regions
- Asymp. Schwarzschild spacetimes with alternating masses M_{BH}, M_{WH}

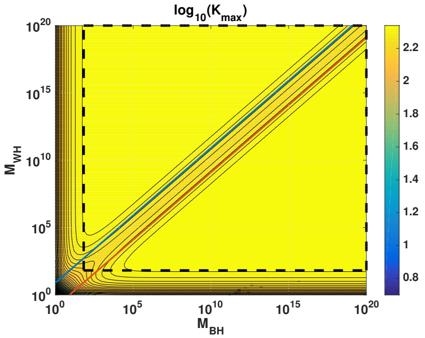
Curvature Upper Bound

Requirement

Unique (mass-indep.) Planckian upper bound for curvature invariants

- Analyse Kretschmann scalar at transition surface
- **All mass relations fine**
- Special is the range

$$\frac{1}{8} < \frac{M_{BH}}{M_{WH}} < 8$$



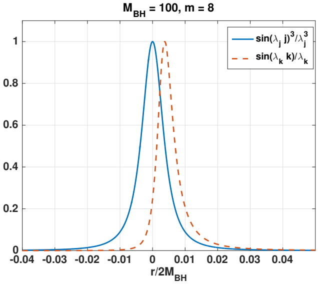
$$\lambda_k = \lambda_j = 1$$

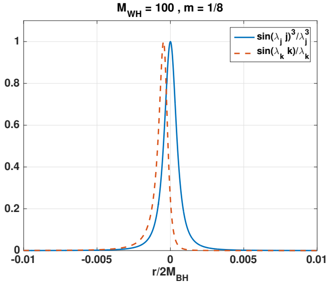
$$M_{WH} = 8 M_{BH} ,$$

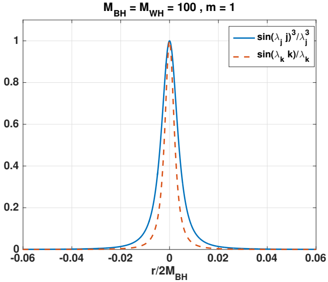
$$M_{WH} = \frac{1}{8} M_{BH} .$$

Onset of Quantum Effects

For $M_{WH} = mM_{BH}$, $\lambda_k = \lambda_j = 1$







- For $\frac{1}{8} < \frac{M_{BH}}{M_{WH}} < 8$ j-sector dominates
- For $m = 1$ only λ_k scale relevant

$$\mathcal{K}_{cl}^{BH} = \frac{48 M_{BH}^2}{\ell_+^6} \ll \frac{3}{4\lambda_k^2} \left(\frac{M_{BH}}{M_{WH}} \right)^2$$

Quantum Theory: A Sketch

[Ashtekar, Corichi, Singh '07; Martín-Benito, Mena Marugán, Olmedo '09]

- For $\lambda_k = \lambda_j = 2$ and $\sqrt{n} = v_j$, we choose the ordering:

$$H_{\text{eff}} = 3\sqrt{v_k}\left(\frac{\sin(2k)}{4}\text{sign}(v_k) + \text{sign}(v_k)\frac{\sin(2k)}{4}\right)\sqrt{v_k}$$
$$\times\sqrt{v_j}\left(\frac{\sin(2j)}{4}\text{sign}(v_j) + \text{sign}(v_j)\frac{\sin(2j)}{4}\right)\sqrt{v_j}$$
$$+\left(\sqrt{v_j}\left(\frac{\sin(2j)}{4}\text{sign}(v_j) + \text{sign}(v_j)\frac{\sin(2j)}{4}\right)\sqrt{v_j}\right)^2 - 2v_j \approx 0$$

- Hilbert space: $|\chi\rangle = \sum_{v_k, v_j \in \mathbb{Z}} \tilde{\chi}(v_k, v_j)\,|v_k, v_j\rangle$

$$\hat{v}_k\,|v_k, v_j\rangle = v_k\,|v_k, v_j\rangle \quad , \quad \widehat{e^{-i\rho k}}\,|v_k, v_j\rangle = |v_k + \rho, v_j\rangle \quad (\rho \in \mathbb{Z})$$

- Rescaling $\tilde{\chi}(v_k, v_j) = \sqrt{|v_k v_j|}\tilde{\psi}(v_k, v_j)$ and changing variables $y_i = \log(\sinh(x_i))$ with $x_1 = \log(\tan(k/2))$ and $x_2 = \log(\tan(j/2))$:

$$\hat{H} = \left(-3\partial_{y_1} - \partial_{y_2} + 4i\cosh(y_2)\right)\partial_{y_2}$$

- Solution of $\hat{H}\,|\psi\rangle = 0$:

$$\psi_{\text{phys}}(y_1, y_2) = g(y_1) + \int^{y_2} dy_2'\, e^{4i\sinh(y_2')}\, f\left(y_2' - \frac{1}{3}y_1\right)$$

Conclusion and Outlook

Summary

- Key role of Dirac observables in effective polymer BH models
- new canonical variables for quantum-corrected Schwarzschild BH (constant polymerisation scales)
- singularity resolved by BH-to-WH transition
- physical viability for all mass relations (symmetric bounce preferred)
- remarkably simple Hamiltonian $\rightarrow$ Quantum theory analytically solvable!

Future work

- complete quantum theory (role of Dirac obs?)
- relation with full LQG
- other spacetimes:
  - gravitational collapse
  - higher dimensions and asymptotically AdS $\rightarrow$ holography?

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Thank you for your attention!

Relation with Connection Variables

Our model in connection variables

Our (v_k, k, v_j, j)-variables are related to connection variables as:

$$p_b^2 = -\frac{8\left(v_k k + v_j j\right)}{j} \quad , \quad |p_c| = 4\, 2^{\frac{2}{3}}\, v_k j \ ,$$

$$b = \text{sign}(p_b)\,\frac{\gamma}{4}\,\sqrt{-8\left(v_k k + v_j j\right) j} \quad , \quad c = -\text{sign}(p_c)\,\frac{\gamma}{4}\,2^{\frac{1}{3}}\,\frac{k}{j} \ .$$

Demanding

$$\lambda_j j \stackrel{!}{=} \lambda_2 P_2 = \delta_b b \qquad , \qquad \lambda_k k \stackrel{!}{=} \lambda_1 P_1 = \delta_c c$$

our model with const. λ's corresponds to the following (*generalised*) $\bar{\mu}$-scheme:

$$\delta_b = \pm\frac{4\lambda_j}{\gamma|p_b|} \qquad , \qquad \delta_c = \pm\frac{64\,2^{\frac{1}{3}}}{\gamma^2}\,\frac{b}{p_b}\,\lambda_k$$