Holography and Unitarity in Gravitational Physics

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UCSB
ILQG Seminar

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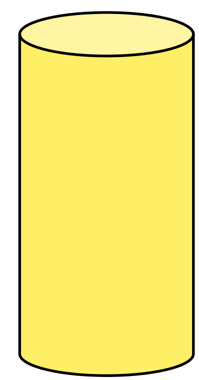
This talk is about:

- Diffeomorphism Invariance and observables in quantum gravity
- The non-locality of quantum gravity observables
- Implications for Information "propagation" (re: Black Holes, AdS/CFT, etc.)

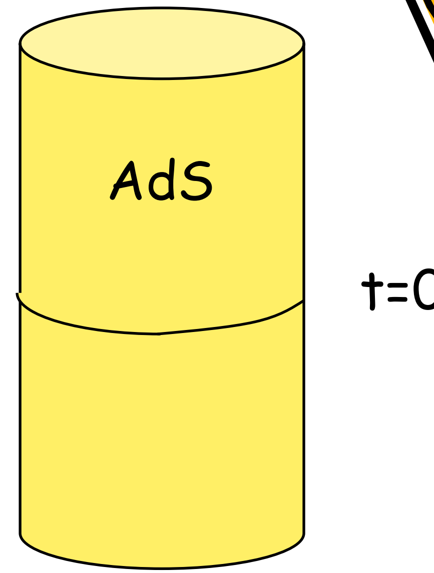
Key Point:

- H_{ADM} is a pure boundary term on the constraint surface.
- ⇒ Clean statements for appropriate boundary conditions.
  - We'll focus on AdS BCs here, or AdS-like BCS, w/ brief comments on As. Flat (details in refs)

    Other BCs = future work



Punch Line: AdS Boundary Unitarity



1. "Boundary Fields" form a natural set of *observables*.
   (E.g., $E_{ab} = C_{abcd}\, n^c n^d$ rescaled and pulled back to bndy.)

   Let $A_{\text{bndy obs}}(t)$ = algebra of boundary observables at time t

2. On the constraint surface, H is a pure boundary term.

   $H = H(t) \in A_{\text{bndy obs}}(t)$

   Suppose this is self-adjoint on *some* Hilbert space.

3. Then H generates time translations (for Observables) via

$$U(t_1, t_2) = \mathcal{P} \exp\left(-i \int_{t_1}^{t_2} H(t)dt\right)$$

⇒ $A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$ "Boundary Unitarity!"

In QM, Information present on the Bndy at any one time t_1 remains present at any other time t_2.

The role of QM

Technical Result:

Similar for QM & CM

$\mathcal{O} \in \mathcal{A}$, generated by A_1, A_2, A_3...

1. Holds in QM with usual notion of algebra
2. Holds classically for Poisson Algebra

Analogy: $J_z \in \mathcal{A}$, generated by J_x, J_y

Physical Interpretation

CM: Measurements of J_x, J_y,... may tell us nothing about J_z!

QM: Information about $\mathcal{O}$ can be obtained by measuring A_1, A_2, ...

(E.g., Suppose an ensemble of identically prepared spins. Find J_z as follows:

For half, measure J_x and then J_y.

For other half, measure J_y and then J_x.)

Outline:

I. Toy Model for AdS: Gravity in a Box

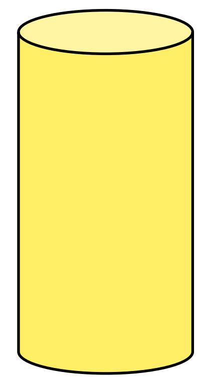
II. Boundary Unitarity

III. Perturbative Holography

IV. AdS Boundary Conditions

V. Comments on As Flat BCs

VI. Summary

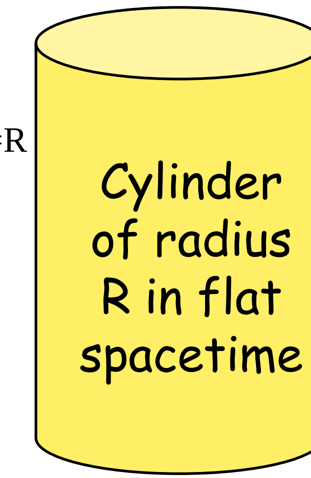


I. Gravity in a box

(*very* similar to AdS BCs)

First, review a familiar problem:
Linear scalar ϕ in a cylinder in flat space time (radius R).

$$\nabla^2\phi - m^2\phi = 0$$



Dirichlet data: $\phi_D = \phi|_{r=R}$ Neumann data: $\phi_N = n^a \partial_a \phi|_{r=R}$

For $z = r-R$, $\phi = \phi_D + z\, \phi_N + O(z^2)$

For well-posed problem, fix one of these (or Robin) as BC.

Key physical point: Good phase space if Klein-Gordon inner product is conserved.

$$F(\delta\phi^1, \delta\phi^2) = \text{KG flux out of cylinder} = \int_{r=R} (\delta\phi_D{}^1\, \delta\phi_N{}^2 - \delta\phi_D{}^2\, \delta\phi_N{}^1)$$

Say, choose Dirichlet BC: maybe $\phi_D = 0$. Then ϕ_N is dynamical.
Call it a "boundary field."

Nothing magic here. ϕ_N is just a part of ϕ, which of course interacts with the other parts of ϕ. Info is continually exchanged btwn ϕ_N and the rest of ϕ.

Same story for non-linear fields.

Gravity in a box

Gravity is similar, too: $G_{ab} = 8\pi T_{ab}$

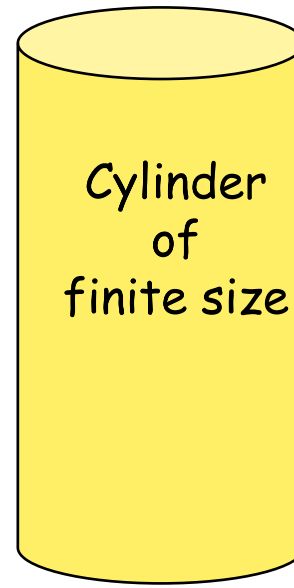
Dirichlet data: $g^{(0)} = g|_{r=R}$ (pull-back)

Neumann data: $P_{ij} = \left[K_{ij} - (1/2)K\, g^{(0)}{}_{ij}\right]|_{r=R}$

Note: In Gaussian Normal Coordinates (w/ z=0 on Bndy)

$ds^2 = dz^2 + g_{ij}(x,z)\, dx^i dx^j$, $x^i = t,x,y.$

$g_{ij} = g_{ij}{}^{(0)} + z\, K_{ij} + \ldots$



Key physical point:
Good phase space if symplectic product is conserved.

$F(\delta g^1, \delta g^2)$ = flux out of cylinder

$$= \int_{Bndy} \left[(\delta g^1)^{ij(0)}\, \delta P_{ij}{}^2 - (\delta g^2)^{ij(0)}\, \delta P_{ij}{}^1 \right]$$

Expect well-posed initial value problem and short-time existence for BCs with F=0. Say, choose Dirichlet BC: fix $g^{(0)}{}_{ij}$.

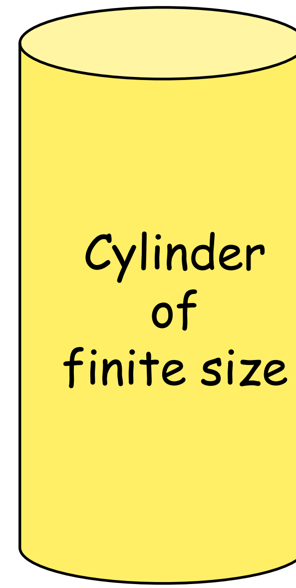
Then P_{ij} is a dynamical "boundary field."

Which Diffeos are Symmetries?

Consider a vector field:

$$\xi^a(x,z) = \xi_{(0)}{}^a\,(x) + z\,\xi_{(1)}{}^a\,(x) + \ldots$$

1) To preserve the cylinder, require $\xi_{(0)}{}^a$ tangent to boundary.



2) Recall: $g_{ij}{}^{(0)}$ is *fixed* as a BC. To preserve BC, $\xi_{(0)}{}^i$ must be a KVF of $g_{ij}{}^{(0)}$.

Note: set of $\xi_{(0)}{}^a$ is finite dimensional.
Defines "asymptotic symmetry group."
Not *gauge* symmetries.

⟹ Gauge symmetries are generated by

$$\xi^a(x,z) = z\,\xi_{(1)}{}^a\,(x) + \ldots$$

These act trivially on Bndy, and leave invariant bndy fields : $\phi_N = n^a \partial_a \phi|_{r=R}$, P^{ij}

I.e., Bndy fields are *observables*.

II. "Boundary Unitarity"

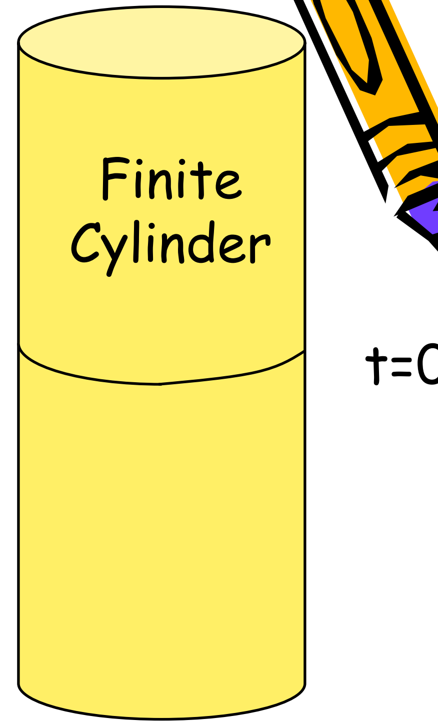
1. "Boundary Fields" form a natural set of *observables*.

   Let $A_{\text{bndy obs}}(t)$ = algebra of boundary observables
   [generated by ϕ_N, P_{ij}] at time t

2. Construct the Hamiltonian:
   On the constraint surface, H is a pure boundary term.
   (Time-dependent of t-trans not a symmetry).

   $H = H(t) \in A_{\text{bndy obs}}(t)$
   [weak equivalence, or action on physical phase space.]

   E.g., for above BCs fixing $g^{(0)}_{ij}$ and $\phi_D=0$, find



$$H(t) := \int_{\text{Bndy Cut w/ t =const}} P_{ij}\, \xi^i\, \underline{n}^i\, dA \qquad \text{(Brown \& York)}$$

with $\xi = \partial_t$ and $\underline{n}^i$ = normal to t= constant cut of boundary.

"Boundary Unitarity," part 2:

$H = H(t) \square A_{\text{bndy obs}}(t)$

E.g., $H(t) := \int_{t=\text{const}} P_{ij}\, \xi^i\, \underline{n}^i \; dA$

Note: For any *observable* $\mathcal{O}$,

$$\partial_t \mathcal{O}(t) = -i\, [\mathcal{O}(t), H(t)]$$

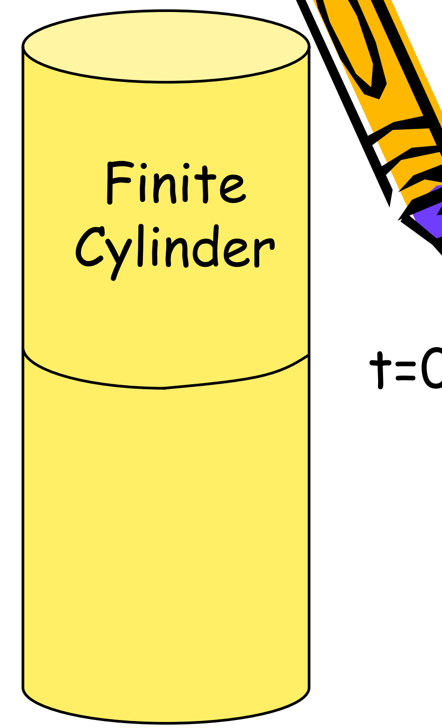
3. *Suppose* * that we can exponentiate H(t) to define

$$U(t_1, t_2) = \mathcal{P} \exp\left(-i \int_{t_1}^{t_2} H(t) dt\right)$$

Then, as in usual QM, find

$$\mathcal{O}(t_2) = U(t_2, t_1)\, \mathcal{O}(t_1)\, U(t_1, t_2)$$

I.e., expresses any Bndy Obs at t_2
in terms of Bndy Fields ϕ_N, P_{ij}, at any other t_1.



⇒ $A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$ "Boundary Unitarity!"

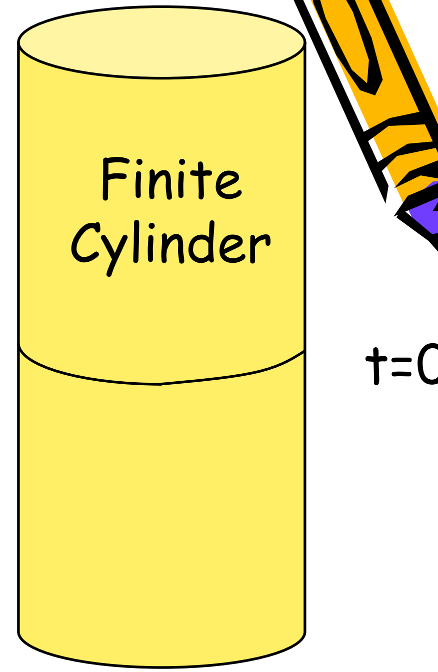
In QM, information present on the Bndy at any one time t_1 remains present at any other time t_2.

Comment on Assumption:

For any *observable* $\mathcal{O}$, $\partial_t \mathcal{O}(t) = -i\,[\mathcal{O}(t), H(t)]$

3. *Suppose* * that we can exponentiate H(t) to define

$$U(t_1, t_2) = \mathcal{P} \exp\left(-i \int_{t_1}^{t_2} H(t) dt\right)$$



Classical Interpretation on space of smooth metrics:

Assumes long-time existence of solutions to EOMs, at least in some neighborhood of the Bndy.

I.e., form of "Cosmic Censorship." (False for finite cylinder.)

QM interpretation:

Assumes quantum Hamiltonian can still be built from ϕ_N, P_{ij}, but that Quantum Gravity "resolves singularities".

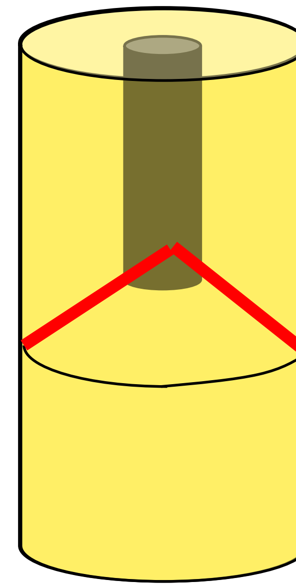
Appears consistent w/ LQG, and easier for BCs where cosmic censorship holds classically.

III. Perturbative Holography

Summary of Above: Any info ever present in the Bndy Fields remains encoded in Bndy Fields.

Q: Is this everything? Or is there more info "in the bulk."

A: Maybe, but "not much."



Consider perturbation theory abt some classical solution which is flat before t=0.

(Though need not remain flat for time-dep BCs.
E.g., can make a black hole.)

At linearized level, any h_{ab}, ϕ can be written (up to gauge) in terms of Bndy observables at early times by solving EOMs.

(Related to Holmgren's Uniqueness Thm.)

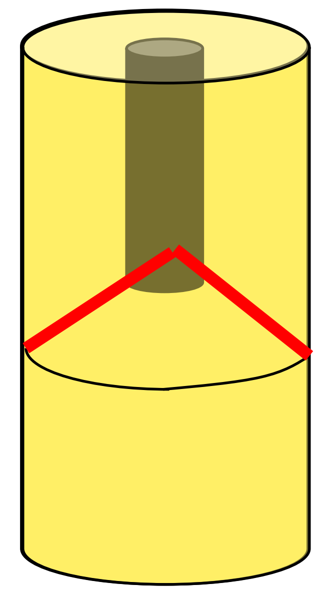
⇒ Remains true at any order in perturbation theory.

Perturbative Holography

So, *any* perturbative observable can be written in terms of Bndy Observables at early times by solving EOMs.

$$A_{\text{All Pert Obs}} = A_{\text{Bndy Obs}}(\text{all } t < 0)$$

But in *gravity*, at any order beyond the linearized theory, the Hamiltonian can again be written as a boundary term!



(I.e., Gauss' Law gives a useful measure of the energy.)

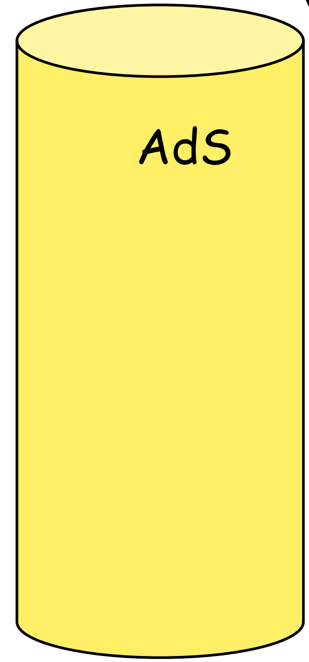
Bndy Unitarity Argument ⇒

$$A_{\text{Pert Obs}} \,(\text{all } t < 0) = A_{\text{Bndy Obs}} \,(\text{any single } t)$$

⇒ $A_{\text{All Pert Obs}} = A_{\text{Bndy Obs}}(\textit{any} \text{ single } t)$

"Perturbative Holography"

IV. Similar story for AdS gravity (D=4)



Fix Bndy @ z=0. In Fefferman-Graham Gauge:

$$ds^2 = z^{-2}\,(dz^2 + g_{ij}(x,z)\, dx^i dx^j), \qquad x^i = t,x,y.$$

Conformal analogue of Gaussian Normal Coordinates.

$$g_{ij} = \underbrace{g_{ij}^{(0)} + z\, g_{ij}^{(1)} + z^2 g_{ij}^{(2)}}_{\text{Determined by } g_{ij}^{(0)}} + z^3 \beta_{ij} + \ldots$$

Determined by $g_{ij}^{(0)}$

Independent of $g_{ij}^{(0)}$ for D=4

Good phase space if symplectic structure is conserved.

$$F(\delta g^1, \delta g^2) = \text{flux out through Bndy} = \int_{\text{Bndy}} [\,(\delta g^1)^{ij(0)}\, \delta T_{ij}{}^2 - (\delta g^2)^{ij(0)}\, \delta T_{ij}{}^1]$$

where $T_{ij} = E_{ij}/(D-3)$, with $E_{ij} = \lim_{z\to 0} z^{D-3}\, C_{ijkl}\, n^k n^l$
is built from $g_{ij}^{(0)}$ and β_{ij}.

E.g, fix $g_{ij}^{(0)}$. Then T_{ij} is a dynamical "boundary field."

Q: Which diffeos are gauge?

A: Only those acting trivially on Bndy
& preserving asymptotic form of "z".

⇨ T_{ij} is an observable. Also true for ϕ_N.

AdS Boundary Unitarity

1. "Boundary Fields" T_{ij}, ϕ_N form a natural set of *observables*.

   Let $A_{\text{bndy obs}}(t)$ = algebra of boundary observables at time t

2. On solutions, H is a pure boundary term.

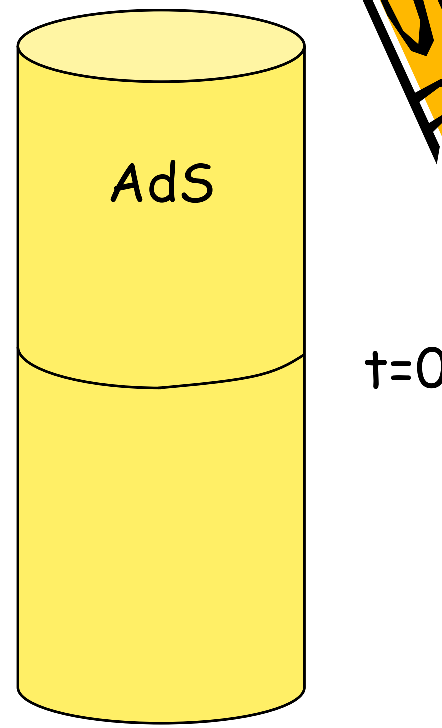
   $H = H(t) \in A_{\text{bndy obs}}(t)$

   Suppose this can be exponentiated.
   Note: Classical cosmic censorship is plausible, especially for AdS_4.

3. Then H generates time translations (for Observables) via

$$U(t_1, t_2) = \mathcal{P} \exp\left(-i \int_{t_1}^{t_2} H(t) dt\right)$$

$\Rightarrow$ $A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$ "Boundary Unitarity!"



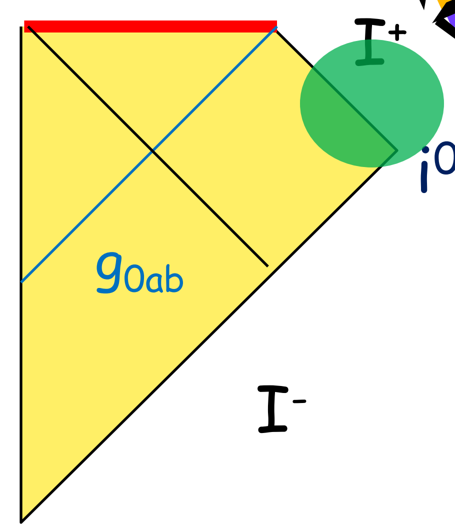
Perturbative Holography also follows, just as for "Gravity in a Box."

V. Comments on As Flat case

1. Perturbative Holography:

   Consider a collapsing black hole background g_{0ab} in pure Einstein-Hilbert gravity.

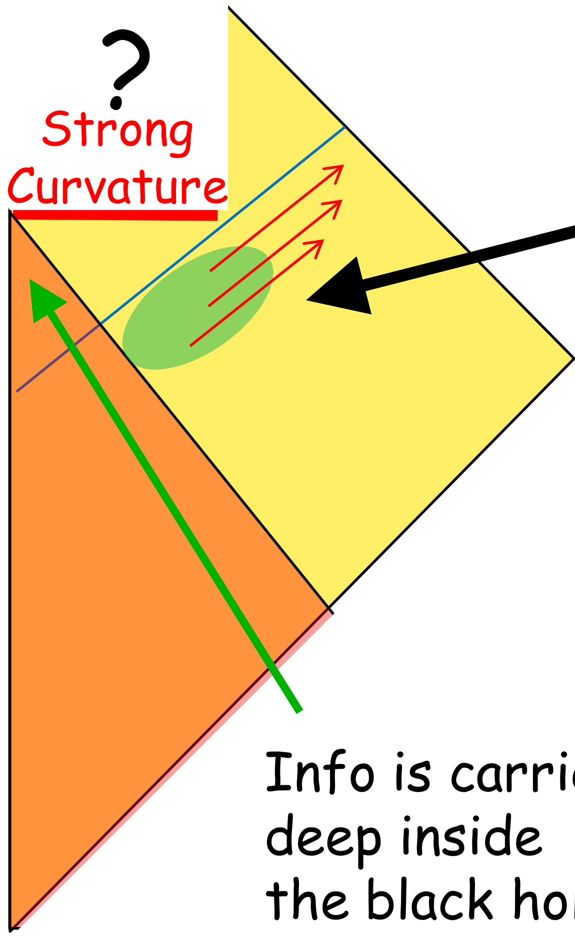
Claim: A complete set of perturbative observables is available on I^+ in any neighborhood of i^0.



2. *Suggests* Unitary S-matrix, with info imprinted in Hawking radiation (next slide).

Basic Mechanism: Constraints and local energy conservation!

Cartoon of BH evaporation



Info is carried deep inside the black hole.

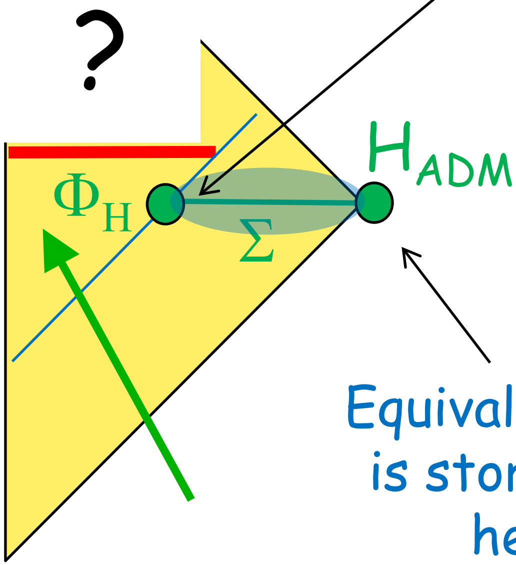
Suppose physics far from strong coupling region is essentially perturbative.

Then perturbative holography implies that all info is encoded in asymptotic fields g_{ab}, especially H_{ADM}.

But constraints relate H_{ADM} to $T^{Hawking}{}_{ab}$ and a surface term "Gauss Law Grav. Flux" Φ_H at the horizon.

$$H_{ADM} - \Phi_H(h) = \int_\Sigma T^{Hawking}{}_{ab}(h)$$

Cartoon of Black Hole Evaporation 2



Remaining info is stored here!

Equivalent info is stored out here

Info carried inside by infalling matter.

$H_{ADM} - \Phi_H(h) \sim \int_\Sigma T_{ab}(h)$

Info shared between Φ_H and T_{ab}.

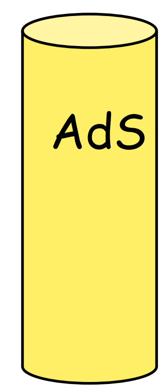
$\Phi_H(h) \to 0$ as BH evaporates.
$\Rightarrow$ info transferred *locally* to T_{ab}.

Indeed, once evaporation is complete, constraint implies $H_{ADM} \sim \int_\Sigma T_{ab}(h)$.

I.e., info fully transferred to Hawking radiation.

Summary: New Perspective

1. Perturbative Holography & (for AdS) Bndy Unitarity follow from gravitational constraints, gauge invariance, and quantum Cosmic Censorship.
2. Info is stored in asymptotic local fields, and throughout BH exterior.
3. Info can be *locally* transferred to Hawking rad via constraints and (local) Energy conservation.



No new causality violation or non-locality required.