Cosmology with group field theory condensates

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The papers so far:

- G., Oriti, Sindoni, *Cosmology from Group Field Theory Formalism for Quantum Gravity*, arXiv:1303.3576 (Phys. Rev. Lett.) and *Homogeneous cosmologies as group field theory condensates*, arXiv:1311.1238 (JHEP)
- G., *Quantum cosmology of (loop) quantum gravity condensates: An example*, arXiv:1404.2944 (Class. Quant. Grav.)
- Calcagni, *Loop quantum cosmology from group field theory*, arXiv:1407.8166 (Phys. Rev. D)
- G., Oriti, *Quantum cosmology from quantum gravity condensates: cosmological variables and lattice-refined dynamics*, arXiv:1407.8167 (NJP)
- Sindoni, *Effective equations for GFT condensates from fidelity*, arXiv:1408.3095
- G., *Perturbing a quantum gravity condensate*, arXiv:1411.1077 (PRD)
- Oriti, Pranzetti, Ryan, Sindoni, *Generalized quantum gravity condensates for homogeneous geometries and cosmology*, arXiv:1501.00936

Related ideas in Calcagni, G., Oriti, *Group field cosmology: a cosmological field theory of quantum geometry*, arXiv:1201.4151 (Class. Quant. Grav.)

Outline

1. Introduction and Motivation
2. Group Field Theory
3. Quantum Gravity Condensates
4. Extracting (Quantum) Cosmology
5. Summary & Outlook

Introduction and Motivation

The cosmology of the very early universe provides the most natural point of contact between theories of quantum gravity and observational phenomena. Practitioners of quantum gravity are hence challenged to

- describe time-dependent, nearly homogeneous and isotropic spacetimes in their approach (assuming the Copernican principle),
- and to provide an effective description for the dynamics of such universes that allows comparison with the predictions of classical GR with inflation (and/or other theories of gravity or cosmological scenarios).

One expects the latter to reduce to something like (semi)classical general relativity when curvature is low, *i.e.* at late times in the evolution of the universe (though this need not be - see 'firewall' discussion for black holes). At very early times large deviations from classical GR are expected.

Introduction and Motivation (II)

Dealing with time-dependent spacetimes is rather challenging in quantum gravity.

Minisuperspace approach [Misner 1969, . . .]: Symmetry reduction at the classical level (to homogeneous, perhaps anisotropic 3-metrics), quantise remaining degrees of freedom. Classical singularity generally not resolved. Successful treatment of perturbations [Halliwell, Hawking 1985, . . .] on 'semiclassical background'.

Loop quantum cosmology (LQC) [Bojowald, Ashtekar, Pawłowski, Singh, Agullo, . . .]: Modified quantisation of FRW background using loop quantum gravity techniques; *big bounce* replaces the singularity. Treat inhomogeneities perturbatively.

Our approach: In the Hilbert space of group field theory (GFT) as a tentative framework for quantum gravity, identify (*condensate*) states that describe homogeneous universes (FRW or Bianchi models); derive *effective* quantum cosmology models from the quantum gravity dynamics of these states.

Two Roads to Quantum Cosmology

Quantum gravity (GFT) model		Classical theory (*e.g.* GR $+ \phi$)
Ansatz for state $\lvert\Psi\rangle$ $\Downarrow$	$\Leftrightarrow$ *Interpretation*	*Ansatz for metric,* ϕ $\Downarrow$
Quantum dynamics for $\lvert\Psi\rangle$ *e.g.* Schwinger–Dyson equations		Classical cosmological dynamics: Friedmann eq. $+$ perturbations
Approximations $\Downarrow$		*'Quantisation'* $\Downarrow$
Effective quantum cosmology		Minisuperspace quantum cosmology
$\mathcal{D}\Psi(g_I) + \text{nonlinearities} = 0$		$\mathcal{H}\Psi(a, \phi, \ldots) = 0$

We can then compare the resulting cosmological dynamics on both sides.

Introduction and Motivation (III)

We work in the *group field theory* (GFT) formalism as a 'second quantised' definition of the kinematics and dynamics of loop quantum gravity (LQG): the *spin foam* amplitudes that can be used to define the dynamics of LQG are generated as Feynman amplitudes of a GFT.

The advantage of using GFT as a standard quantum field theory language for LQG is the availability of tools and intuition from other areas of physics. In particular, we use (generalised) *coherent states* of the GFT field that are similar to states used to describe **Bose–Einstein condensates**. These states are our proposal for approximating a continuum non-degenerate phase of LQG/GFT.

We can consider different types of suitable states; in the simplest approximation they describe a 'gas' of weakly interacting 'atoms' or 'molecules'. Continuum space(time) emerges in the *hydrodynamic approximation* of the condensate, as a *quantum fluid*. The states involve superposition of an infinite number of graphs.

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Group Field Theory

Basic idea: Group field theories define a path integral for discrete quantum gravity, including a sum over the topologies and discretisations (triangulations) of spacetime, for a given (discrete) boundary geometry.

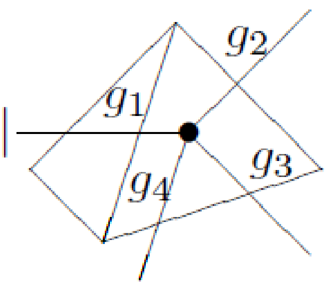
Boundary states are elements of the GFT Fock space, and labelled by the arguments of the GFT field $\varphi(g_I)$, a complex field on four copies of a Lie group G. Group elements are the possible parallel transports associated to four links (dual to the faces of a tetrahedron) or equivalently to a 4-valent open LQG spin network vertex.

$$Z = \int \mathcal{D}\varphi \, \mathcal{D}\bar{\varphi} \; e^{-S[\varphi,\bar{\varphi}]} = \sum_{\Gamma} \frac{\lambda^N}{\mathrm{sym}[\Gamma]} Z(\Gamma)$$

is then, for appropriate choice of S, a tentative sum-over-histories for quantum gravity in 4d (not defined on a fixed lattice or discretisation!). Quantum field theory not *on* but *of* space: many standard QFT techniques available!

Group Field Theory (II)

The GFT vacuum describes a completely degenerate geometry with zero volumes, areas, etc.; excitations over it are elementary tetrahedra with geometric data:

$$\hat{\varphi}^\dagger(g_1, g_2, g_3, g_4)|\emptyset\rangle = |$$  $$\rangle$$

The interpretation of g_I is as parallel transports of the gravitational connection, $g_I \sim \mathcal{P} \exp \int A$. The dual ('momentum') variables represent a (discrete) metric.

This tetrahedron is dual to a 4-valent vertex of an LQG spin network; with $G = \mathrm{SU}(2)$ arbitrary 4-valent spin network states can be identified with many-particle states in the GFT Fock space [Oriti 2013]. The Fock space defines the *kinematical* Hilbert space of GFT on which dynamics must be imposed.

Group Field Theory (III)

Dynamics for group field theory is defined by a choice of action,

$$S[\varphi,\bar{\varphi}] = \int (\mathrm{d}g)^4\ (\mathrm{d}g')^4\ \bar{\varphi}(g_I)\mathcal{K}(g_I,g'_I)\varphi(g'_I) + \mathcal{V}[\varphi,\bar{\varphi}]$$

which can be chosen such that its Feynman amplitudes equal the quantum amplitudes of a *spin foam model* [Reisenberger, Rovelli 2001].

The potential $\mathcal{V}$ defines the 'gluing' of tetrahedra to form 4-dimensional structures (the LQG vertex amplitude) while the kinetic term defines the propagator. This propagator is normally chosen to be trivial for spin foam models; demanding *renormalisability* of the GFT action suggests that a non-trivial propagator including derivatives may be required [Ben Geloun, Bonzom, Carrozza, Oriti, Rivasseau, . . .].
One may also consider a more general class of potentials, *e.g.*, including quantum corrections.

Group Field Theory (IV)

In LQG we aim to directly 'quantise' general relativity, *i.e.* formulate a consistent quantum theory starting from the variables of a classical theory of gravity, with microscopic dynamics derived from a variant of the Einstein–Hilbert action.

From other situations in quantum field theory and other approaches to quantum gravity, we know that the quantum theory can have low-energy properties that are not present classically (*e.g.* mass gap in QCD, nongeometric phases in CDT).

In a quantum field theory setting such as GFT, one can study collective properties of a large number of degrees of freedom using *renormalisation* methods. One may expect to find, depending on the coupling constants of the theory, different ground states describing different *phases*, such as a **geometric** phase describing an approximate smooth 4-dimensional spacetime, or a **non-geometric** phase with very different properties. These phases are typically described by inequivalent Hilbert spaces.

Outline

Quantum Gravity Condensates

The natural notion of GFT vacuum is the Fock vacuum $|\emptyset\rangle$ which corresponds to the Ashtekar–Lewandowski vacuum in LQG. This vacuum describes a completely degenerate phase in which areas, volumes etc are all zero.

We suggest the existence of a different phase in GFT, away from this Fock vacuum, which is akin to the *condensation* of a large number of bosons into a common ground state in condensed matter physics. We follow the analogy with real Bose–Einstein condensates in approximating the (physical) state describing this phase by a *coherent state* of the GFT field operator $\hat{\varphi}(g_I)$ with $\langle\varphi\rangle \neq 0$.

The condensate can be interpreted geometrically as a macroscopic homogeneous universe. We define it as a state in the GFT Fock space with a large number N of quanta; for an actual phase transition one would work in the thermodynamic limit $N \to \infty$, which plays the role of continuum limit, and would correspond to a different Hilbert space (normalisability in the Fock space requires $N < \infty$).

Quantum Gravity Condensates (II)

We describe a 'condensation' of many GFT quanta into the same microscopic quantum state by states analogous to coherent states for Bose–Einstein condensates, or squeezed states in quantum optics. The simplest states are of the form

$$|\sigma\rangle = \mathcal{N}(\sigma)\exp\left(\int (\mathrm{d}g)^4\,\sigma(g_I)\,\hat{\varphi}^\dagger(g_I)\right)|\emptyset\rangle\,,$$

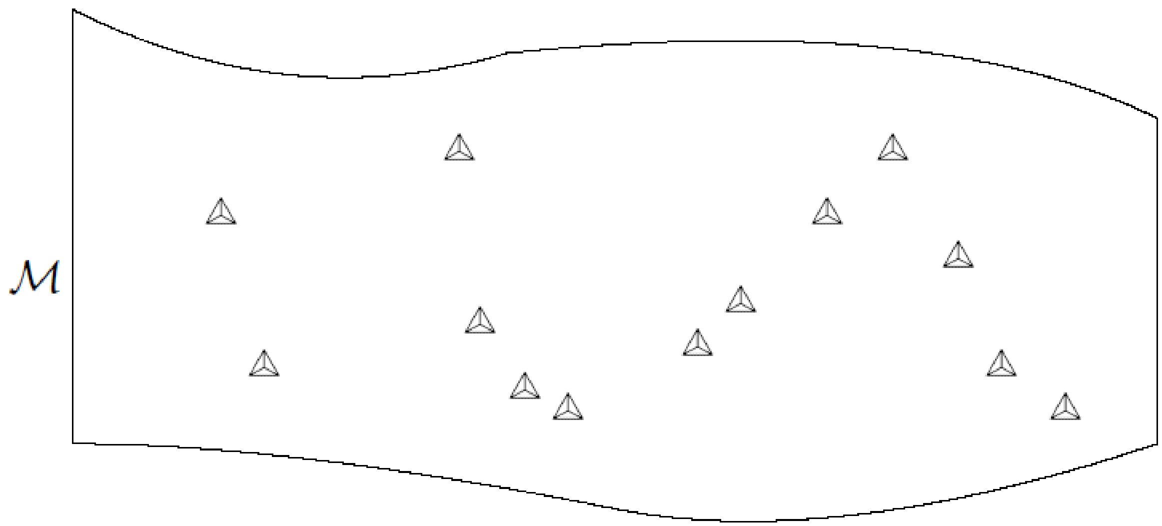
which can be viewed as a 'gas' of tetrahedra all in a state determined by the 'condensate wavefunction' $\sigma(g_I)$. There are constraints on σ to ensure that it only depends on gauge-invariant data.

$$|\xi\rangle = \mathcal{N}(\xi)\exp\left(\int (\mathrm{d}g)^4\,(\mathrm{d}h)^4\,\xi(g_I^{-1}h_I)\,\hat{\varphi}^\dagger(g_I)\hat{\varphi}^\dagger(h_I)\right)|\emptyset\rangle$$

gives a gas of 'dipoles', *i.e.* pairs of tetrahedra with all faces pairwise identified, which form triangulated 3-spheres. From the LQG viewpoint, these states include a superposition of graphs with different numbers of vertices.

Quantum Gravity Condensates (III)

To justify the geometric interpretation of these states, we use an embedding into a manifold $\mathcal{M}$ with a group acting on it (defining the notion of homogeneity).



Using a frame of the left-invariant vector fields to fix the orientation of the tetrahedra, one finds that the geometric information contained in the state corresponds to a spatial metric which is (in this frame) the same at N points. We superpose all different N; continuum emerges in the **hydrodynamic** description.

Coherent States for Bose–Einstein Condensates

This construction is analogous to using a coherent state to characterise a Bose–Einstein condensate

$$|\Psi\rangle = \mathcal{N}(\Psi)\exp\left(\int \mathrm{d}^3x\ \Psi(\vec{x})\hat{\Phi}^\dagger(\vec{x})\right)|\emptyset\rangle\,,$$

given in terms of the 'condensate wavefunction' $\Psi(\vec{x})$. Ψ has a direct physical interpretation: writing it as

$$\Psi(\vec{x}) = \sqrt{\rho(\vec{x})}\,e^{-\mathrm{i}\theta(\vec{x})}$$

the fields $\rho(\vec{x})$ and $\theta(\vec{x})$ can directly be interpreted as the density and velocity potential (*i.e.* $\vec{v}\propto\vec{\nabla}\theta$) of the fluid represented by the condensate.
The **hydrodynamic** equations for the *classical* fields $\rho(\vec{x})$ and $\theta(\vec{x})$ emerge as an approximation to the *quantum dynamics* for Ψ (the Gross–Pitaevskii equation).

Quantum Gravity Condensates (IV)

We now proceed similarly in the case of GFT.
One can derive an effective dynamics for the 'condensate wavefunction' $\xi(g_I)$ from the Schwinger–Dyson equations

$$\left\langle \frac{\delta \mathcal{O}[\varphi,\bar{\varphi}]}{\delta\bar{\varphi}(g_I)} - \mathcal{O}[\varphi,\bar{\varphi}]\frac{\delta S[\varphi,\bar{\varphi}]}{\delta\bar{\varphi}(g_I)}\right\rangle_\xi = 0$$

for some observable $\mathcal{O}[\varphi,\bar{\varphi}]$ and the GFT action S. $\mathcal{O}=1$ gives the Gross–Pitaevskii equation for the BEC. (Alternative approach using *fidelity* [Sindoni 2014])
In general this takes the form of nonlinear, nonlocal differential equations for $\xi(g_I)$.

The state is determined by the 'condensate wavefunction' $\xi(g_I)$. ξ is a complex function on the possible configurations of a homogeneous universe (made up of identical tetrahedra), *i.e.* a *minisuperspace*. It is the quantum cosmology analogue of the order parameter $\Psi(\vec{x})$ in real condensates.

Extracting (Quantum) Cosmology

As an approximation to the full GFT dynamics, we focus on the simplest Schwinger–Dyson equations which become differential equations for the 'wavefunction' $\xi(g_I)$. Just like the 'condensate wavefunction' in condensed matter physics, $\xi(g_I)$ does not have the probability interpretation of an actual wavefunction. It rather corresponds to a field on minisuperspace that encodes the hydrodynamic description of the GFT condensate.

One can identify (global) condensate observables with cosmological variables (scale factor, Hubble 'parameter' etc) or quantum numbers with no classical analogue (*e.g.* particle number). Such observables are expectation values of operators such as

$$\hat{\mathcal{O}} := \int (\mathrm{d}g)^4 \; \hat{\varphi}^\dagger(g_I) O(g_I) \hat{\varphi}(g_I)$$

analogous to the total momentum, energy or a global charge for a real condensate. The Schwinger–Dyson equations give relations between these expectation values.

Extracting (Quantum) Cosmology (II)

In the simplest approximation, one can show that one of the Schwinger–Dyson equations reduces to

$$\int (\mathrm{d}g')^4 \, \mathcal{K}(g_I, g'_I)\xi(g'_I) = 0$$

where $\mathcal{K}$ is the kinetic operator of the GFT action, so that one obtains a linear equation in ξ only depending on the quadratic part of the GFT action.

As one example, taking $\mathcal{K} = \sum_I \Delta_{g_I} + m^2$ and for an isotropic universe, computing expectation values gives a semiclassical 'Friedmann' equation

$$\frac{1-\sin^2(\mu\,\omega)}{a^2} - \frac{\mathrm{i}\, l_{\mathrm{Pl}}^2}{a^4} N^{2/3} \left(\frac{2}{\sin(\mu\omega)} - 3\sin(\mu\omega)\right) - \frac{m^2 l_{\mathrm{Pl}}^4 N^{4/3}}{2a^6} = 0.$$

We see the appearance of *holonomy corrections* as in LQC with $\mu \propto N^{-1/3}$. Cosmological interpretation depends on $N = N(a)$ (*cf.* work on lattice refinement in LQC)! We recover the improved dynamics of LQC for $N \propto a^3$.

Extracting (Quantum) Cosmology (III)

A classical 'Friedmann equation' arises in the hydrodynamic approximation of the full GFT dynamics for condensate states. Summary of main results so far:

- Formal (WKB) $l_{\rm Pl} \to 0$ limit: Friedmann equation for vacuum GR, in either Lorentzian or Riemannian signature and including a massless scalar; using matter coupling, LQC holonomy corrections can be derived [Calcagni 2014];
- taking the scaling with N into account and $l_{\rm Pl} > 0$, effective Friedmann equations depend on the relation $N = N(a)$, which can (in principle) be derived;
- form of LQC holonomy corrections $\mu \propto N^{-1/3}$ can be derived from considerations of intensive/extensive observables in the GFT Fock space;
- wide phenomenology: possible emergence of effective cosmological constant, corrections to GR, etc., from a given proposal for quantum gravity dynamics.

Extracting (Quantum) Cosmology (IV)

So far the main focus in cosmological applications has been on understanding the semiclassical limit in regions far away from high curvature. Major questions for (quantum) cosmology can also be addressed in this programme:

- What is the fate of the initial singularity, or more generally, is there a breakdown of the approximation in which a condensate approximates well a fully dynamical state? As for the BEC, such a breakdown of the approximation at high curvature might signal a GFT *phase transition*. Alternatively, the condensate might be a good description in the Planckian regime (as in LQC);

- How can we depart from exact homogeneity? We need to include small perturbations which can be localised and described as effectively propagating on the homogeneous 'background' condensate. Inhomogeneities may appear as nonlinearities in quantum cosmology [Bojowald et al. 2012] – our quantum cosmology equations are naturally nonlinear.

Summary & Outlook

- *Condensates* of GFT 'atoms' or 'molecules' are a proposal for describing a macroscopic, spatially homogeneous non-degenerate universe in the context of GFT as a formalism for LQG. In the simplest approximation these states describe a 'gas' of disconnected quanta but one can also work on connected graphs that include topological information [Oriti et al. 2015; see Sindoni's ILQGS].

- Using the GFT formalism, we can treat such states with methods similar to those used in Bose–Einstein condensates. The effective dynamics of the 'condensate wavefunction' defines an effective quantum cosmology model.

- A (generalised) Friedmann equation emerges in the hydrodynamic description for expectation values of GFT observables.

- **Open questions:** Consistency of low-energy description; examination of stability of approximations (fate of singularity?); role of GFT phase transitions; treatment of anisotropic and inhomogeneous metrics; matter fields; Λ; . . .

Thank you!