

Why study boundaries?

Important physics happens at boundaries

- Generic presence of **edge/surface/boundary/corner** **modes/charges/states/waves**
- In gauge theories (e.g. gravity) **boundaries** support **charges** and **symmetry algebras**
- Boundaries may be at ∞ (e.g. J^+), finite distance (BH), “fake” (entanglement), . . .

Fits the logic of LQG

- Take seriously the **classical structure of general relativity**
- Build a quantization of GR which puts **symmetries at the center** (Diff, SU(2) so far)
- Start with **quasi-local** data and **build spacetime from gluing and coarse-graining**

New opportunities for LQG

- Can reconcile LQG (bulk-based) with “holographic” (boundary-based) approaches
- Possible tests of the classical underpinnings (Barbero–Immirzi parameter and tetrads)
- LQG tools are well-suited for the quantization of the boundary symmetries

Many tools already available

- Tensors networks and holography, boundary states, boundary dual theories, . . .
- [Bianchi, Bodendorfer, Campiglia, Carrozza, Chirco, Colafranceschi, Delcamp, Dittrich, Livine, Goeller, Han, Hoehn, Hung, Husain, Oriti, Perez, Riello, Sengupta, Speziale, Steinhaus, Thiemann, Varadarajan, Wen, Zhang, . . .]

What classical GR tells us

Charges and symmetries

- In gauge theories, Noether's theorems assign codimension-2 charges to symmetries

$$\delta Q_\xi = \delta_\xi \cdot \Omega = \delta Q_{\text{Noether}} + Q_{\text{flux}}$$

- Evolution of a charge

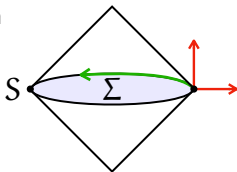
$$\delta_\xi Q_\zeta = Q_{[\xi, \zeta]} + \delta_\zeta \cdot Q_{\text{flux}}$$

evolution = rotation + dissipation

- Corner symmetry group

$$G_S = (\text{Diff}(S) \ltimes H) \ltimes \mathbb{R}^2$$

group = kinematical \ltimes dynamical



Formulation-dependent

- For a given formulation \$F\$ of gravity, the symplectic structure is

$$\Omega_F = \Omega_{\text{ADM}} + d\Omega_{F/\text{ADM}}$$

- Different formulations have different symmetry groups \rightarrow **inequivalent quantizations**

ADM	H =	\emptyset
Einstein–Hilbert	H =	$SL(2, \mathbb{R})_\perp$
Einstein–Cartan	H =	$SL(2, \mathbb{R})_\parallel \times (\text{Boosts})$
Einstein–Cartan–Holst	H =	$SL(2, \mathbb{R})_\parallel \times SL(2, \mathbb{C})$

Some implications

Discreteness at the classical and continuum level

- LQG symplectic structure

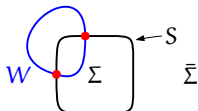
$$\Omega_{\text{LQG}} = \Omega_{\text{ADM}} + d(\delta \mathbf{E}_I \delta \mathbf{n}^I - \gamma \delta \mathbf{e}_I \wedge \delta \mathbf{e}^I)$$

- Fluxes \mathbf{E}_I form the familiar $\mathfrak{su}(2)$ algebra of LQG
- Tangential metric $q_{ab} = e_a^I e_b^J \eta_{IJ}$ on S forms an $\mathfrak{sl}(2, \mathbb{R})$ algebra

$$\{q_{ab}(x), q_{cd}(y)\} = -\gamma(q_{ac}\epsilon_{bd} + q_{bc}\epsilon_{ad} + q_{ad}\epsilon_{bc} + q_{bd}\epsilon_{ac})(x)\delta^2(x-y)$$

- Casimirs related by $C_{\text{SL}(2, \mathbb{R})} = -(\gamma^{-1}\sqrt{q})^2 = C_{\text{SU}(2)} \rightarrow$ **quantization of area element**

$$\sqrt{q}(x) = \gamma \ell_{\text{Pl}}^2 \sum_i \sqrt{j_i(j_i + 1)} \delta^2(x - x_i)$$



Symmetries control the behavior of subsystems

- Subsystem can be subregion (how to cut / glue ?) or whole spacetime (“holography”)
- The logic of LQG tells us to label states with reps. of the boundary symmetry group

