Quantum Evaporation of 2-d Black Holes

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CGHS Black holes: A conceptually new angle and high precision numerics.

This talk will put together and summarize results obtained over past 2 years.

I. Introduction   II. Mean Field Approximation   III. Full Quantum Gravity

Joint work with: Victor Taveras and Madhavan Varadarajan (Analytical issues), and,

Franz Pretorius and Fethi Ramazanoglu (Numerical issues); Discussions with Amos ori.

International Loop Quantum Gravity Seminar, September 21st, 2010

I. Introduction

• Standard black hole evaporation scenario assumes (Hawking, '74)

i) As in the classical theory, space-time is asymptotically flat at future null infinity $\mathcal{I}^+$ also in full quantum gravity;
ii) Because of quantum radiation the Bondi mass $M_{\rm B}$ decreases and reaches the Planck size when the horizon area shrinks to Planck size;
iii) Process is quasi-static during this long phase. Therefore the radiation $\mathcal{I}^+$ is thermal;

Then, correlations are continuously lost over this very long phase. At the end, black hole has a Planck scale mass and full quantum gravity becomes essential for the first time. But whatever the details of its predictions, there is too little left over mass to compensate the enormous loss of correlations.

$\Rightarrow$ The S-matrix cannot be unitary.

Pure state on $\mathcal{I}^-$ evolves to a mixed state on $\mathcal{I}^+$. There is information loss.

• Solid as this argument seems, it ignores the possibility that small secular quantum gravity corrections may invalidate the Hawking scenario (AA, Bojowald). We will see that assumptions ii) and iii) are in fact violated in the quantum evaporation of 2-d black holes.

I. What is new

- Viewpoint: Consider the Callen-Giddings-Harvey-Strominger 2-d BHs with emphasis on similarities and differences with spherical 4-d BHs in GR. De-emphasize the extremal dilatonic BHs or c=1 non-critical strings which provided the original motivation.

- New Formulation: The ‘ mathematical state’ defined on a fiducial geometry is easy to compute both classically and quantum mechanically; non-triviality lies in its interpretation in the *physical* geometry. (AA, Taveras, Varadarajan (ATV))

- Mean Field Approximation (MFA): Recent analysis shows that prior intuition on the semi-classical sector (large N limit) needs important corrections. High precision numerics leads to an unforeseen universality. Interesting in its own right particularly for geometric analysis. (AA, Pretorius, Ramazanoglu (APR))

- Full Quantum Gravity: A space-time picture of how information is recovered. Intuition on singularity resolution derived from Loop Quantum Gravity. But the analysis much more general and not tied to LQG. (ATV)

- Status: Excellent analytic and numerical control on the MFA/semi-classical gravity. Using these MFA results, together with three educated assumptions, we conclude that the S matrix is unitary and discuss the physics of the outgoing state.

I. Classical collapse of a scalar field

• Spherical collapse of a scalar field f in 4-d GR:

Writing ${}^4g_{ab} = g_{ab} + r^2\, s_{ab} \equiv g_{ab} + \frac{e^{-2\phi}}{\kappa^2}\, s_{ab}$, the action reduces to

$$S(g,\phi,f) = \frac{1}{2G} \int d^2x \sqrt{|g|}\, \left[e^{-2\phi}\,(R + 2\nabla^a\phi\nabla_a\phi + 2e^{-2\phi}\kappa^2) + Ge^{-\phi}\nabla^a f\nabla_a f\right]$$

• The 2-d Callen-Giddings-Harvey-Strominger (CGHS) Black hole:

$$S(g,\phi,f) := \frac{1}{2G} \int d^2x \sqrt{|g|}\, \left[e^{-2\phi}\,(R + 4\nabla^a\phi\nabla_a\phi + 4\kappa^2) + G\,\nabla^a f\nabla_a f\right]$$

f: scalar field; Setting $g^{ab} = \Omega\,\eta^{ab}$, gravitational sector: (ϕ, Ω).

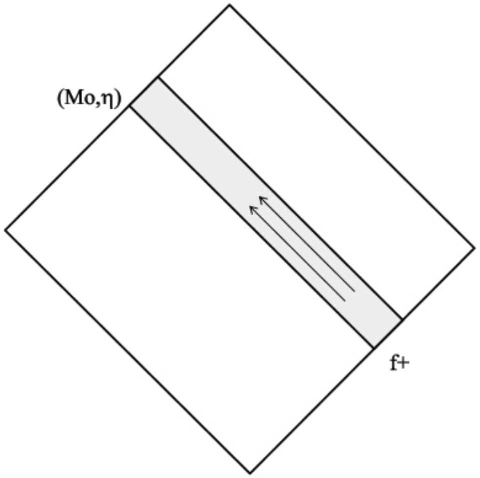
• 4-d and 2-d rather similar but CGHS is technically much simpler: for, the matter field f now satisfies $\Box_\eta\, f = 0$, and, given any solution f we can write down the solution for ϕ, Ω in a closed form algebraically !

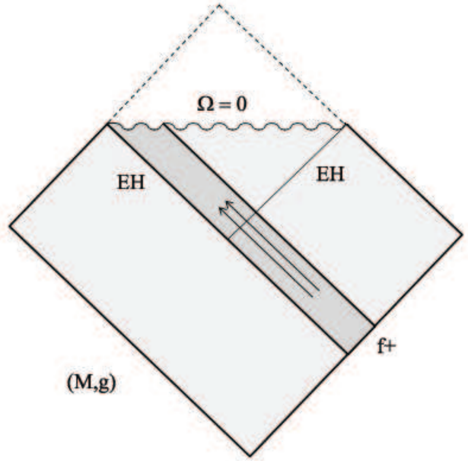
Setting $\eta_{ab} = -\partial_{(a} z^+\, \partial_{b)} z^-$, $\kappa\, x^\pm = \pm e^{\pm\kappa z^\pm}$, $\Phi = e^{-2\phi}$, $\Omega = \Theta^{-1}\Phi$

The solution is: $f = f_+(z^+) + f_-(z^-)$, $\Theta = -\kappa^2 x^+\, x^-$ and

$$\Phi = \Theta - \frac{G}{2}\int_0^{x^+} d\bar{x}^+ \int_0^{\bar{x}^+} d\bar{\bar{x}}^+\, (\partial f_+/\partial\bar{\bar{x}}^+)^2 - \frac{G}{2}\int_0^{x^-} d\bar{x}^- \int_0^{\bar{x}^-} d\bar{\bar{x}}^-\, (\partial f_-/\partial\bar{\bar{x}}^-)^2$$

I. Gravitational collapse in 2-d: CGHS solution





• Start with a general solution $f_+(z^+)$ in Minkowski space (M_0, η).
It determines a full solution Φ, Θ and $g^{ab} = \Phi\Theta^{-1}\eta^{ab}$.
These fields —the classical state— regular everywhere on M_0.

• How can there be a black hole, then?

• Φ vanishes along a space-like line. Curvature of g blows up there. So physical space-time (M, g) is smaller. But $\mathcal{I}_R^+$ is complete: The affine parameter y^- on $\mathcal{I}_R^+$ w.r.t. (M, g) runs from $-\infty$ to ∞. And past of $\mathcal{I}_R^+$ is not all of M. $\Rightarrow$ Black hole!
BH emerges when one interprets the mathematically trivial solution f, Φ, Θ on fiducial (M_o, η), using the physical geometry g.

I. Hawking radiation

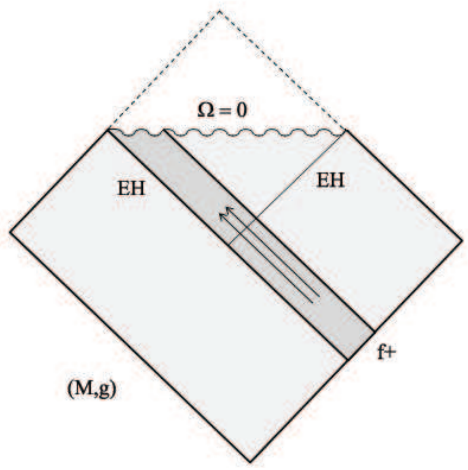
• Black hole formed by the gravitational collapse of some left moving modes $f_+(z^+)$ which fall in from $\mathcal{I}_R^-$. Consider the right moving mode as a test quantum field $\hat{f}_-(z^-)$ on this dynamical BH space-time. Since $f_- = 0$ on $\mathcal{I}_L^-$, natural to assume that $\hat{f}_-(z^-)$ is in a vacuum state on $\mathcal{I}_L^-$. What is the outgoing state on $\mathcal{I}_R^+$ for this test field $\hat{f}_-$ on the dynamical BH background space-time (M, g)?

• $\hat{f}_-$ Dynamics trivial on the entire fiducial space-time (M_o, η): $\Box_{(\eta)}\hat{f}_- = 0$. On $\mathcal{I}_R^{o+}$, state is just the vacuum state $|0_R^+\rangle$ in the $\pm$ freq. decomposition w.r.t. inertial observers $\partial/\partial z^-$ of η.

• Interpretation w.r.t. (M, g)?
i) Need $\pm$frequency decomposition
w.r.t. inertial observers $\partial/\partial y^-$ of (M, g); and,
ii) trace over modes on $\mathcal{I}_R^{o+} - \mathcal{I}_R^+$.

Result: On (M, g), we have a thermal state at temperature $\kappa\hbar$! Information loss!

(Unlike in 4-d, temperature independent of mass.)



II. Back reaction: Mean field approximation

• Framework for full quantum theory exits (Later in the talk). Mean Field Approximation (MFA): Ignore the quantum fluctuations of geometry, i.e., of $(\hat{\Theta}, \hat{\Phi})$ but not of matter $\hat{f}$. Requires a large number N of matter fields $\hat{f}$. PDEs for $\langle\hat{\Phi}\rangle := \Phi$ and $\langle\hat{\Theta}\rangle := \Theta$ but now they include back-reaction.

• Hyperbolic evolution Eqs:

$\Box_{(\eta)}\, f = 0 \quad \Leftrightarrow \quad \Box_{(g)} f = 0$

$\partial_+\,\partial_-\,\Phi + \kappa^2\Theta = G\hbar\bar{N}\,\partial_+\,\partial_-\,\ln\Phi\Theta^{-1} \equiv G\,\langle\hat{T}_{+-}\rangle$ (conformal anomaly)

$\Phi\partial_+\,\partial_-\ln\Theta = -G\hbar\bar{N}\,\partial_+\,\partial_-\,\ln\Phi\Theta^{-1} \equiv -G\,\langle\hat{T}_{+-}\rangle$ (conformal anomaly)

(where $\bar{N} = N/24$)

• Constraint Eqs imposed at $\mathcal{I}^-$ (and preserved in time):

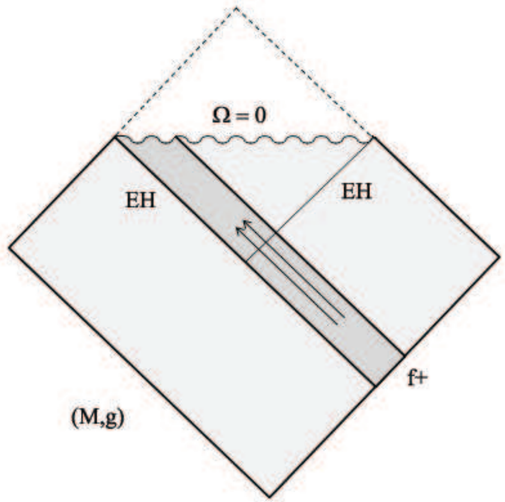
$-\partial_-^2\,\Phi + \partial_-\,\Phi\partial_-\,\ln\Theta = G\,\langle\hat{T}_{--}\rangle \hat{=} 0$

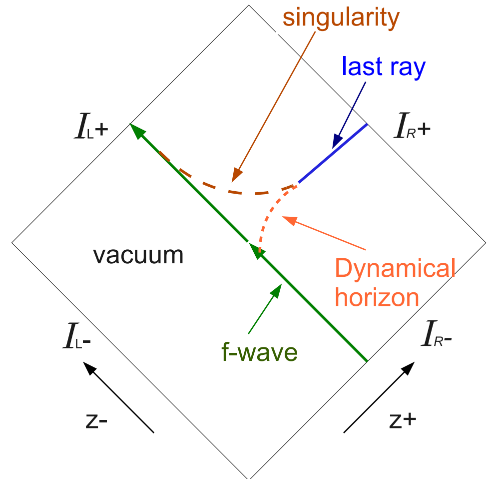
$-\partial_+^2\,\Phi + \partial_+\,\Phi\partial_+\,\ln\Theta = G\,\langle\hat{T}_{++}\rangle \hat{=} \Theta(z^\pm) - 12\bar{N}GN\int_0^{x^+}\mathrm{d}\bar{\mathrm{x}}^+\int_0^{\bar{\mathrm{x}}^+}\mathrm{d}\bar{\bar{\mathrm{x}}}^+\,(\partial\mathrm{f}_+/\partial\bar{\bar{\mathrm{x}}}^+)^2$

• Physical metric in MFA: $g^{ab} = \Phi\,\Theta^{-1}\eta^{ab}$.

Task: Solve these equations. Global Issues: Do these solutions g^{ab} admit $\mathcal{I}_R^+$? Is the Bondi mass on $\mathcal{I}_R^+$ positive? What is it's value at the end of the MFA evaporation? Positive? Negative? Large? Planck scale? ...

II. Answers: Analytic results & high precision numerics



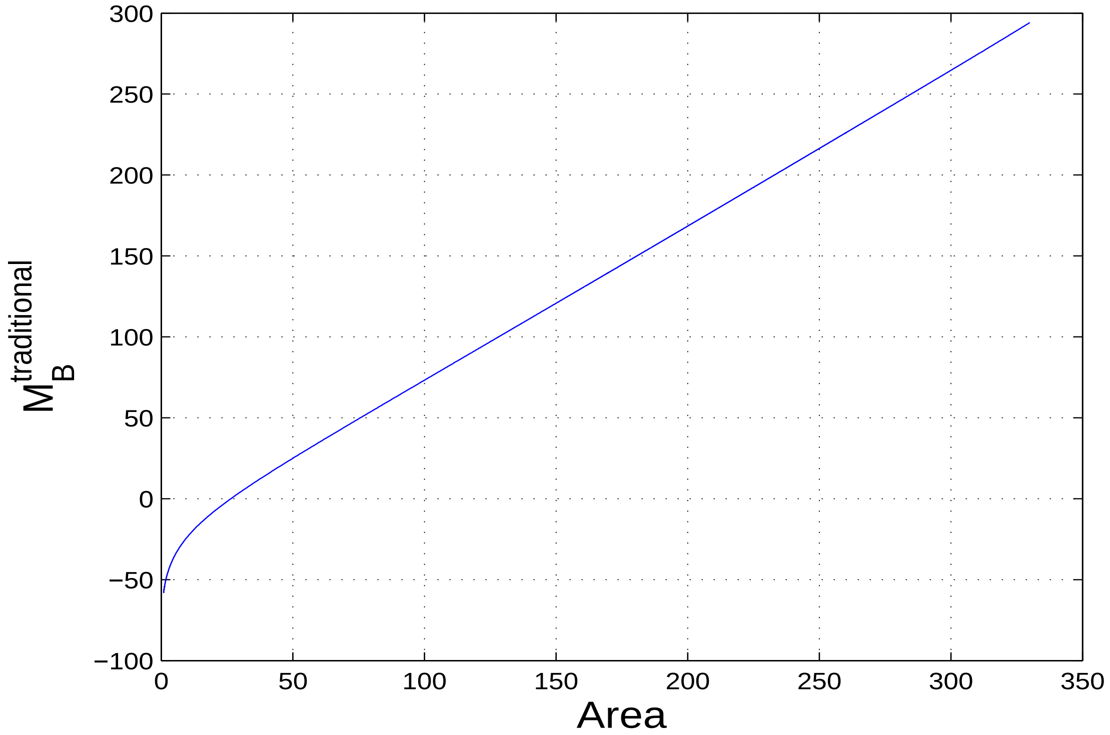


• Numerics $\Rightarrow$ g is asymptotically flat at right future null infinity $\mathcal{I}_R^+$.

• Space-like singularity persists in MFA. But it is weak; g is invertible & C^0 but not C^1 there. Also ends because of evaporation; does not reach $\mathcal{I}^+$ as in the classical space-time! Last ray: future boundary of the MFA space-time. Furthermore, Ricci scalar finite at the last ray: the singularity does not propagate out to infinity.

• How big is $\mathcal{I}_R^+$? Numerics: The affine parameter w.r.t. the physical metric g is finite at the last ray of the MFA space-time, as hoped. (Otherwise information would be definitely lost!) $\mathcal{I}_R^+$ is incomplete $\Rightarrow$ no event horizon in semi-classical space-time. What dorms and evaporates is the dynamical horizon.

II. Traditional Bondi mass becomes negative!



Simulations: Fethi Ramazanoglu

Time dependence of the traditional Bondi mass (Susskind et al, Hayward, ...) $M_{\rm B}^{\rm T}$ is plotted against the area of the dynamical horizon for $M_{\rm ADM} = 360$ and $N = 720$. During evaporation $M_{\rm B}^{\rm T}$ becomes negative even when the horizon area is macroscopic; larger the N, more negative it becomes.

II. ATV Bondi mass and flux at $\mathcal{I}_R^+$

• Not surprising from 4-d GR perspective: Traditional definition of Bondi-mass $M_{\rm B}^{\rm T}$ taken from the classical static solutions (Susskind et al, Hayward, ...). Analogy in 4-d: Taking the Bondi mass to be $M_{\rm B} = \oint {\rm d}^2{\rm V}\, \Psi_2^0$ —rather than $\oint {\rm d}^2{\rm V}\, (\Psi_2^0 + \bar\sigma^0 \dot\sigma^0)$ — also in dynamic situations. But then $M_{\rm B}$ or its flux would not be positive in 4-d dynamical situations.

• But Following the 4-d Bondi procedure, a new expression had already been proposed 2 years ago, using the balance law (ATV):

Asymptotically, $\Phi = A(z^-)e^{\kappa z^+} + B(z^-) + O(e^{-\kappa z^+})$ near $\mathcal{I}_R^+$.

A determines the affine parameter y^- on $\mathcal{I}_R^+$ w.r.t. g, and, B satisfies

$$\frac{\rm d}{{\rm d}y^-}\Big[\frac{\rm dB}{{\rm d}y^-} + \kappa B \;+\; \bar N \hbar G \,\Big(\frac{{\rm d}^2 {\rm y}^-}{{\rm dz}^{-2}}\,\big(\frac{{\rm dy}^-}{{\rm dz}^-}\big)^{-2}\Big)\Big] = -\frac{\bar N \hbar G}{2}\,\Big[\frac{{\rm d}^2{\rm y}^-}{{\rm dz}^{-2}}\,\big(\frac{{\rm dy}^-}{{\rm dz}^-}\big)^{-2}\,\Big]^2$$

ATV definition: $M_{\rm B}(y^-) \;=\; \frac{\rm dB}{{\rm dy}^-} + \kappa B + \bar N \hbar G\,\Big(\frac{{\rm d}^2{\rm y}^-}{{\rm dz}^{-2}}\,\big(\frac{{\rm dy}^-}{{\rm dz}^-}\big)^{-2}\Big)$

• Then: i) The definition agrees with that in the static case; ii) The flux is manifestly negative; iii) When it vanishes, $\partial/\partial y^- = \partial/\partial z^-$ at $\mathcal{I}_R^+$. Furthermore, iv) Numerics $\Rightarrow$ $M_{\rm B}$ is positive all the way to the last ray!

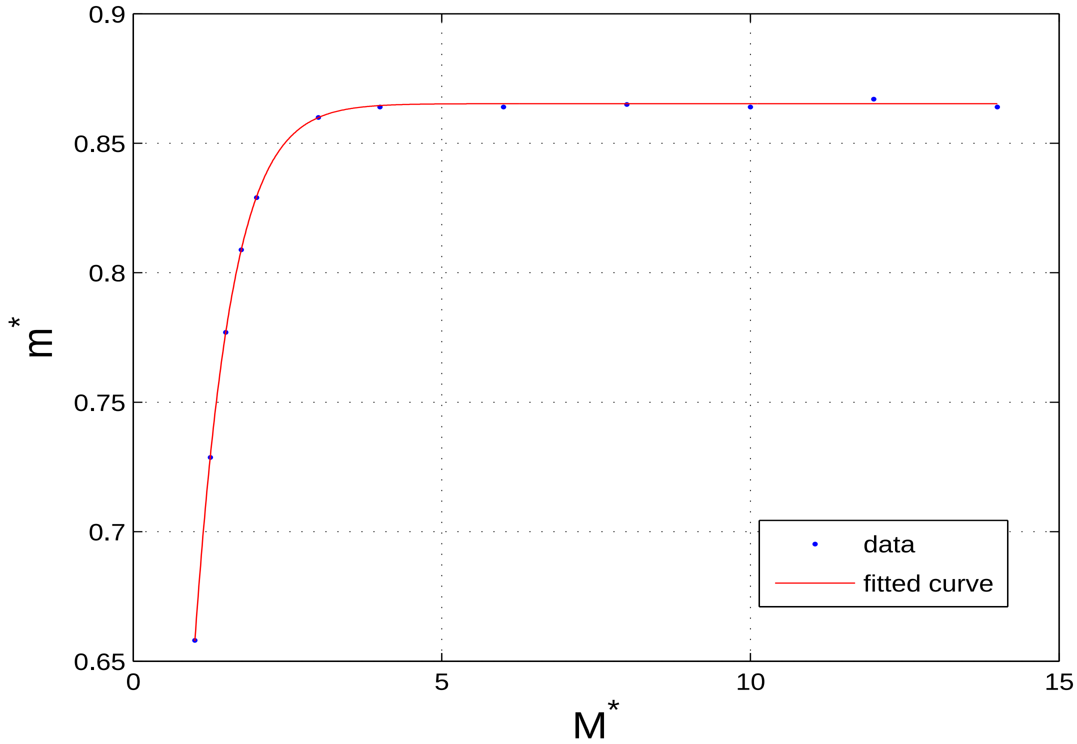
II. Scaling symmetry and Universality

• A New Realization (inspired by numerics!): If $(f, N, \Theta, \Phi,)$ is a solution to MFA equations,
so is $(f, \lambda N, \lambda\Theta, \lambda\Phi)$ for any real constant λ.
Under this scaling, $g \to g, \; M_{\rm ADM} \to \lambda M_{\rm ADM} \;\; M_{\rm B} \to \lambda M_{\rm B}$.
So, for geometry, energetics, interpretation at $\mathcal{I}_R^+$, etc what matters are dimensionless quantities, e.g., $M^\star = (M_{\rm ADM}/\bar{N}\hbar\kappa)$, and
$m_{\rm B}^\star = (M_{\rm B}/\bar{N}\hbar\kappa)$.

• Numerics $\Rightarrow$ Universal Behavior:
i) Global Process: For Macroscopic BHs, $m^\star := m_{\rm B}^\star|_{\text{(last ray)}}$ is universal:
$m^\star \approx 0.86$ in Planck units.

ii) Dynamics: The ATV-Bondi flux is zero at early times and rises quickly once the trapped surface is formed. After this transient phase, the curve joins a universal curve. Thus for macroscopic BHs, the evolution at $\mathcal{I}_R^+$ is universal.

II. Universality: Masses



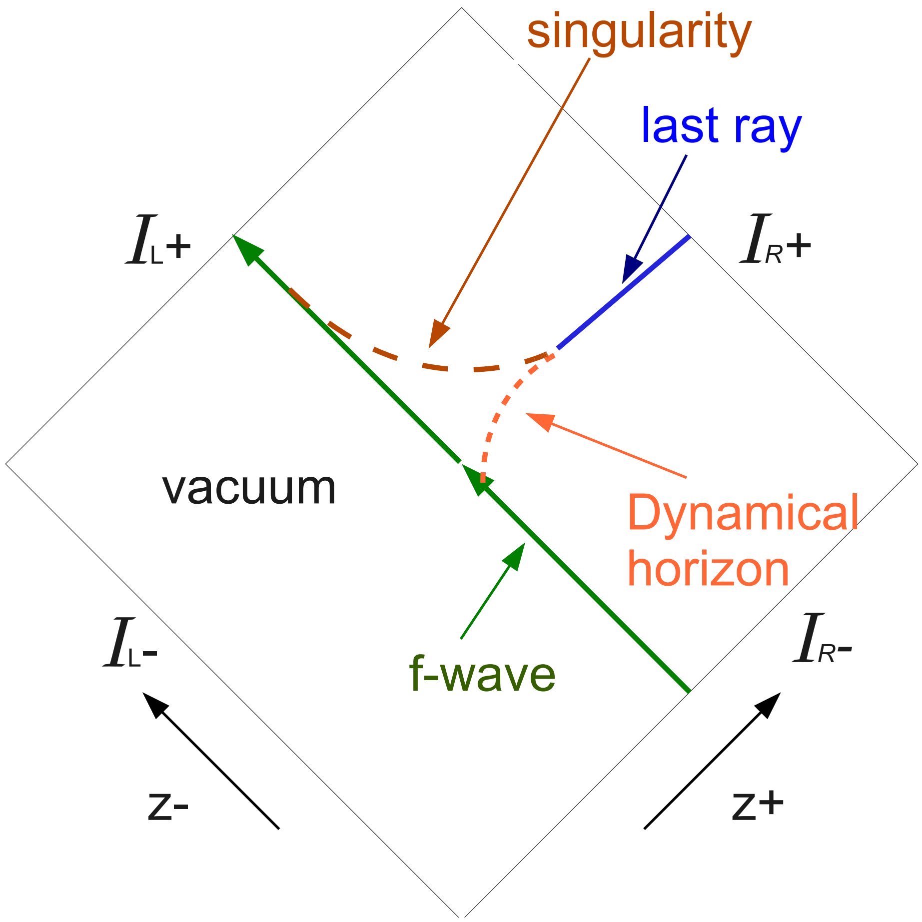
Simulations: Fethi Ramazanoglu

$m^\star = 24 M_{\rm B}/\kappa\hbar N$ at the last ray as a function of $M^\star = 24 M_{\rm ADM}/\kappa\hbar N$ for $M* = 14, 12, 10, 8, 6, 4, 3, 2, 1.75, 1.50, 1.25, 1$. Data points fitted to the curve $m^\star = \alpha\,(1 - e^{-\beta (M^\star)^\gamma})$ with $\alpha \approx 0.86,\ \beta \approx 1.42,\ \gamma \approx 1.15$.

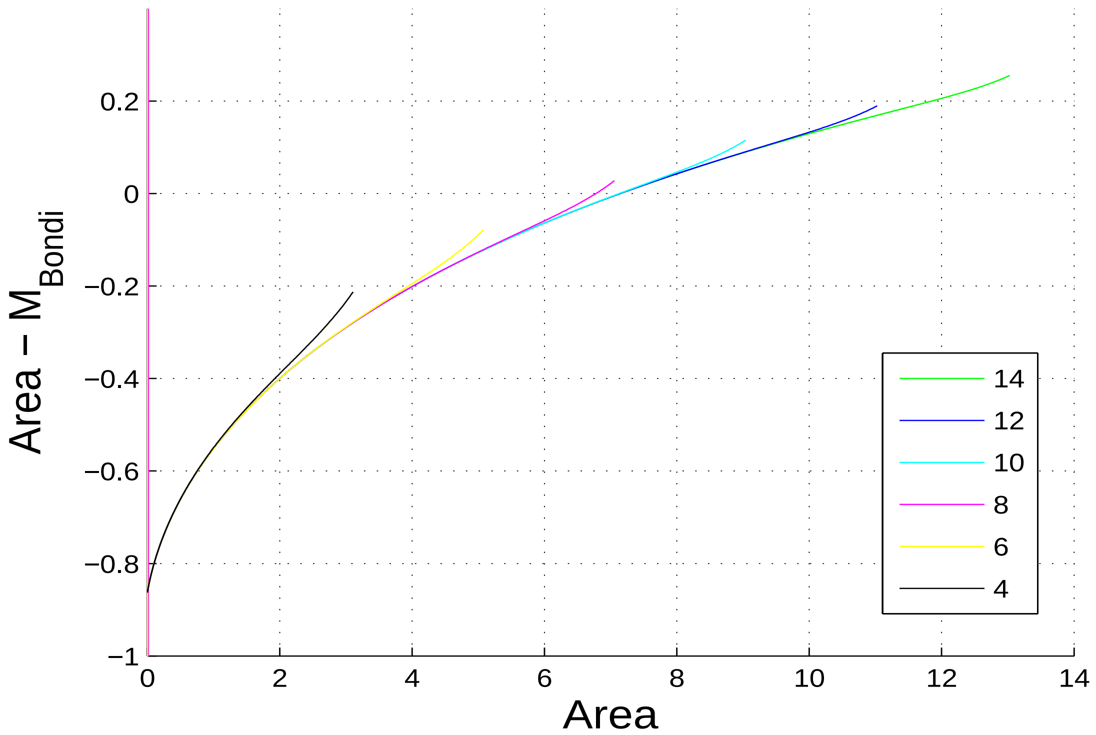
Sharp transition: Macro/micro BHS around $M^\star = 3$.

(Piran-Strominger $M^\star = 1$; Lowe: $M^\star = 1.25$.)

MFA: schematic Penrose diagram



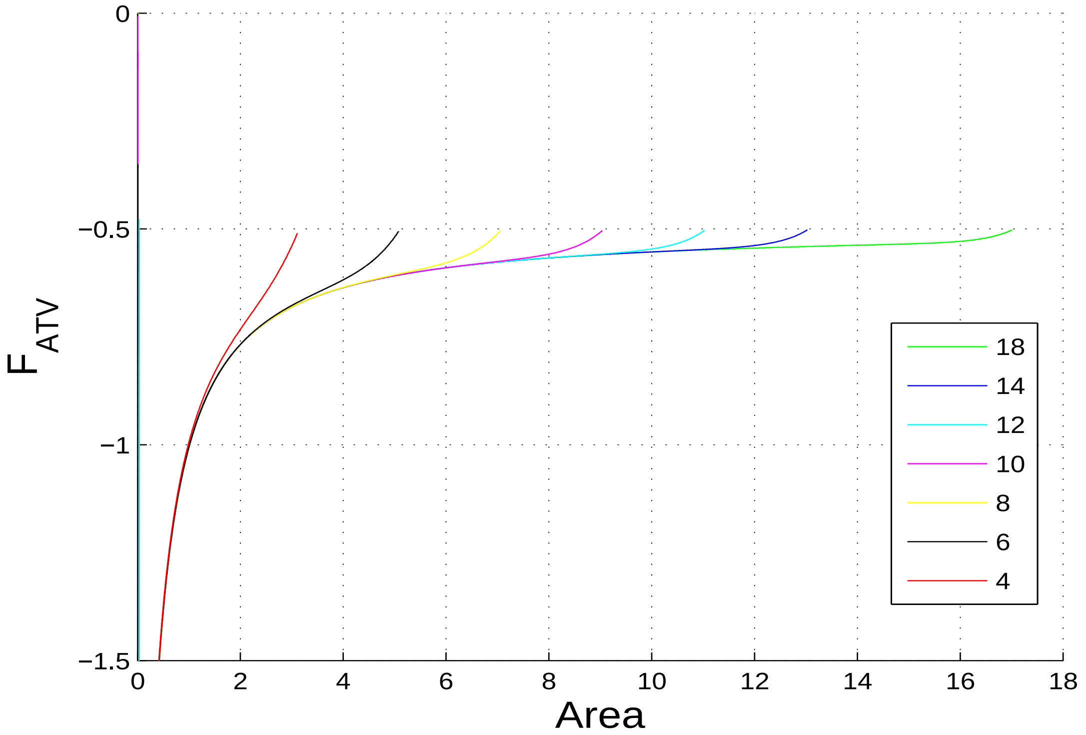
III. Universality: Dynamics



Simulations: Fethi Ramazanoglu

ATV Bondi-mass as a function of of the area of the dynamical horizon. At the end of the transient epoch, the curves for different ADM masses $M^\star$ join a universal curve.

II. Universality: Dynamics



Simulations: Fethi Ramazanoglu

The ATV Bondi-flux as a function of the area of the dynamical horizon. For each ADM mass $M^\star$ the flux rises very quickly in a transient region immediately after the formation of the dynamical horizon and approaches a universal curve. The Hawking/thermal flux of the external field approximation is a constant (0.5 all these BHS) $\Rightarrow$ steady departure from a thermal flux over a long time.

II. Universality: Summary

• Numerics $\Rightarrow$ Universal behavior:
i) Global Process: For Macroscopic BHs, at the last ray, i.e. end of the MFA evolution, $m^\star \approx 0.86$ in Planck units.
ii) Dynamics: The ATV-Bondi flux is zero at early times and rises quickly once the trapped surface is formed. After this transient phase, the curve joins a universal curve. Thus for macroscopic BHs, the evolution at $\mathcal{I}_R^+$ is universal. Interesting problems for mathematical relativity/geometric analysis.

• However, there is a small but cumulative difference between this MFA evolution (which includes back reaction) and external field approximation of Hawking effect (which does not). $\Rightarrow$ Even the flux is not thermal. This is important for the recovery of information.

III. Full quantum gravity: Framework

- On the fiducial flat background, solve for operator-valued distributions $\hat{f}, \hat{\Phi}$ and operator $\hat{\Theta}$ satisfying:

- Hyperbolic evolution Eqs:

$$\Box_{(\eta)}\, \hat{f} = 0$$

$$\partial_+\,\partial_-\,\hat{\Phi} + \kappa^2\hat{\Theta} = \tfrac{G\hbar}{24}\,\partial_+\,\partial_-\,\ln\hat{\Phi}\hat{\Theta}^{-1}; \quad \hat{\Phi}\partial_+\,\partial_-\ln\hat{\Theta} = -\tfrac{G\hbar}{24}\,\partial_+\,\partial_-\,\ln\hat{\Phi}\hat{\Theta}^{-1}$$

- Boundary conditions at $\mathcal{I}^-$:

$$-\partial_-^2\,\hat{\Phi} + \partial_-\,\hat{\Phi}\partial_-\,\ln\hat{\Theta} \hat{=} 0$$

$$-\partial_+^2\,\hat{\Phi} + \partial_+\,\hat{\Phi}\partial_+\,\ln\hat{\Theta} \hat{=} \Theta(z^\pm) - \tfrac{G}{2}\int_0^{x^+} \mathrm{d}\bar{\mathrm{x}}^+ \int_0^{\bar{\mathrm{x}}^+} \mathrm{d}\bar{\bar{\mathrm{x}}}^+\,(\partial\hat{\mathrm{f}}_+/\partial\bar{\bar{\mathrm{x}}}^+)^2$$

Should be possible to solve these equations at least perturbatively in the dimensionless Planck number $G\hbar$. True DOF in $\hat{f}$. So $\hat{\Phi}, \hat{\Theta}$ will be defined on the Fock space of $\hat{f}$. Initial state: vacuum $|0\rangle_L^-$ for $\hat{f}_-$ on $\mathcal{I}_L^-$ and coherent state $|\Psi\rangle_R^-$ for $\hat{f}_+$ on $\mathcal{I}_R^-$

- We know that, in the fiducial geometry of η, the state $|0\rangle_L^-$ on $\mathcal{I}_L^-$ 'evolves' to $|0\rangle_R^+$ on $\mathcal{I}_R^+$. : Question: What is the interpretation of this mathematical state in the physical geometry $\hat{g}$ constructed from $\hat{\Phi}, \hat{\Theta}$?

III. Full quantum gravity: Framework

- Three assumptions:

i) The mathematical solutions $\hat{f}, \hat{\Phi}, \hat{\Theta}$ are well-defined on M_o (supporting evidence exits);
ii) MFA holds near $\mathcal{I}_R^{o+}$ (standard assumption); and, iii) After the ‘last ray’, the remainder Bondi-mass, ~ 0.86 per scalar field, will be emitted to $\mathcal{I}_R^+$ and after that the Bondi mass and Bondi flux would vanish at $\mathcal{I}_R^+$ (common assumption).

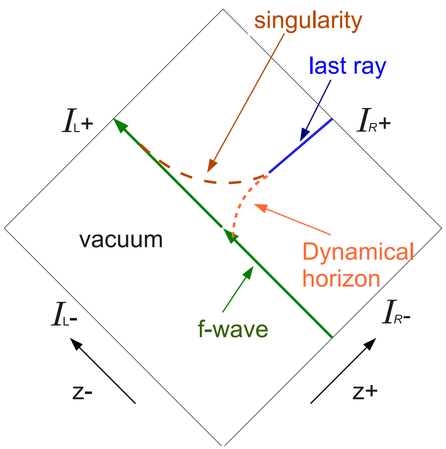
- Key questions:

i) Is $\mathcal{I}_R^+$ of the physical geometry $\bar{g} := \langle \hat{g} \rangle$ equal $\mathcal{I}_R^{o+}$ of η or a proper subset of $\mathcal{I}_R^{o+}$ as in the MFA,

If $\mathcal{I}_R^+ \neq \mathcal{I}_R^{o+}$, we would have to trace over some modes and in the physical geometry $\bar{g}$, the ‘out state’ will be a mixed state $\Rightarrow$ S-matrix will fail to be unitary.

ii) Even if $\mathcal{I}_R^+ = \mathcal{I}_R^{o+}$ question remains:
what is the relation between the affine parameters y^- and z^- of $\mathcal{I}_R^+$ w.r.t. $\bar{g}$ and η?

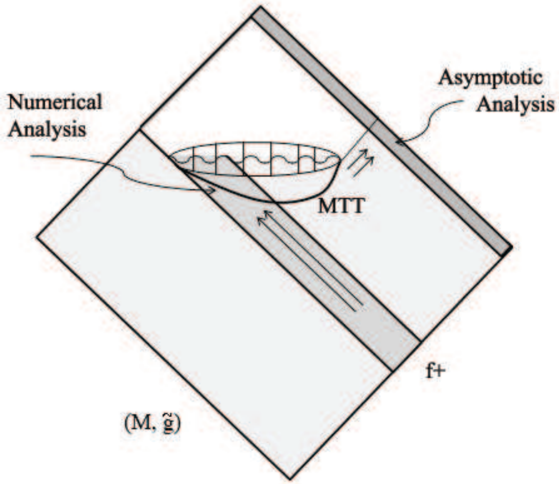
If they agree asymptotically as one approaches i^o and i^+, $\pm$ frequency decomposition w.r.t. y^- and z^- will be unitarily equivalent $\Rightarrow$ S matrix will be unitary. Otherwise not.



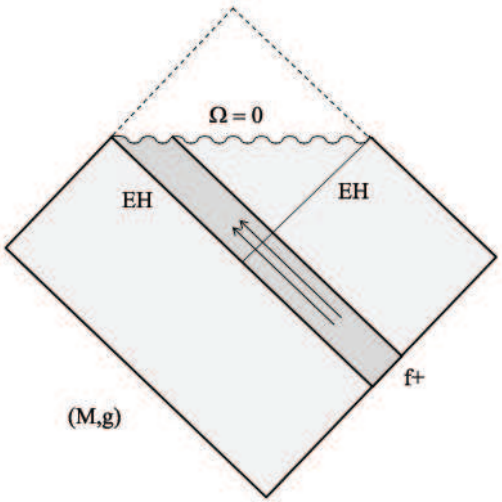
III. Results & the ATV Scenario

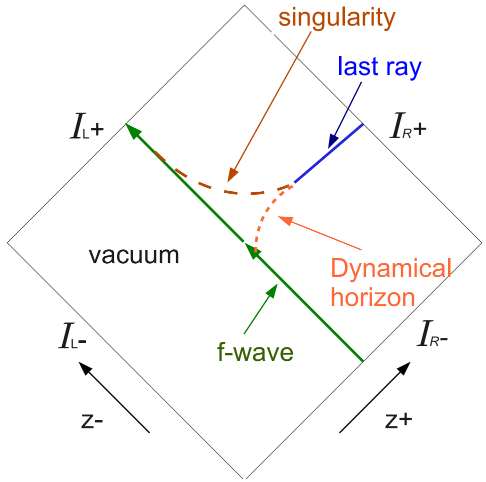
• Information not lost because detailed analysis shows that our assumptions imply:
i) $\mathcal{I}_R^+ = \mathcal{I}_R^{o+}$; no tracing over modes; and,
ii) y^- related to z^- in such a way that the Bogoluibov transform is well defined; i.e., the vacuum state at $\mathcal{I}_R^+$ w.r.t. η does belong to the Fock space at $\mathcal{I}_R^+$ of the physical geometry g.

• Space-time description: Singularity would be replaced by a 'quantum region' where the quantum geometry is fine but the MFA/large N approximation fails because of very large quantum fluctuations. (supporting evidence: mini-superspace analysis (Ori); truncated theory (ATV); Resolution of 4-d BH singularity in LQG (AA, Bojowald; ...) .



Summary: Classical and Semi-Classical Theory





• In the classical theory, the mathematical state f, Φ, Θ, is regular everywhere on M_o. Black hole arises when it is interpreted in the physical metric g.

• Hawking effect in the dynamical, BH space-time: Again the state of $\hat{f}_-$ is just $|0\rangle$ on (M_o, η). But on (M, g) it is interpreted as a thermal state with temperature $\kappa\hbar$.

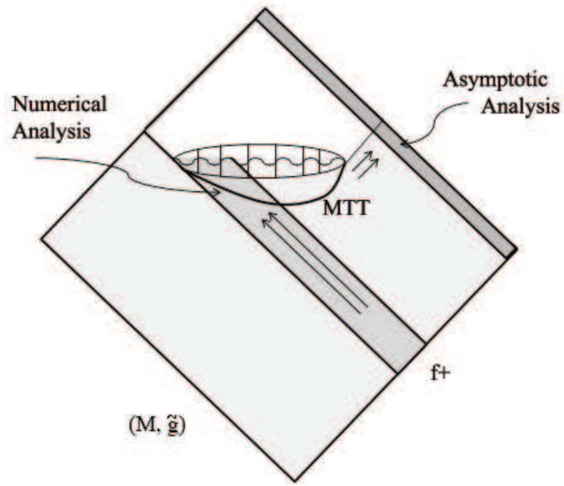
• Back reaction is included in the MFA/large N approximation. Singularity persists but weak. The traditional Bondi mass becomes negative and increases with N. The ATV Bondi-mass remains positive. High precision numerics shows unanticipated universality for a large number of physical quantities. For all initially macroscopic BHs, at the end of the MFA space-time, $m^\star = 0.86$ remains to be evaporated per scalar field.

Summary: Quantum Theory

• Full Quantum Theory: Under 3 assumptions, we are led to the ATV scenario in full QG. State on $\mathcal{I}_R^+$ is pure & belongs to the asymptotic Hilbert space of the physical metric $\bar{g}$ $\Rightarrow$ the S-matrix is unitary. No information loss.

• When does the information come out? To the extent the question is well-defined, 'most of it comes out before the last ray'. (Very little energy carried away by each scalar field after the end of the semi-classical space-time.) Possible because the state is not really thermal during evaporation; even the energy-flux is different from that of thermal radiation over a long period.

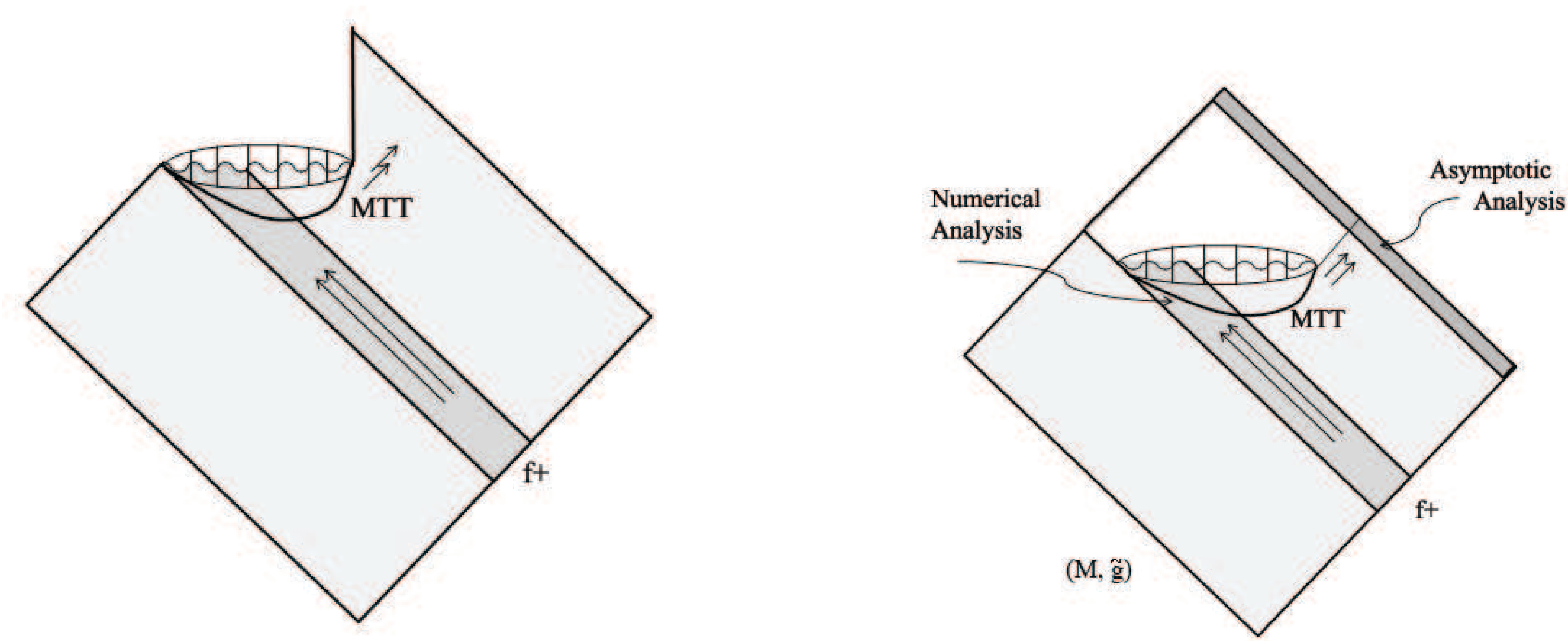
• While great many conceptual similarities with the 4-d picture, there are also some important differences:
1) In 2-d, the Hawking Temperature does not depend on M.
2) In CGHS, dynamics of f and geometry are is decoupled.
3) Scri has two disconnected pieces. Collapsing f_+ moves to the left while f_- quantum radiation moves to the right.



Somewhat surprisingly:
In the shell-collapse, the f_+ information is also recovered on $\mathcal{I}_R^+$! Don't know if this is also the case more generally.

Summary: In a Nutshell

The old and the new Penrose diagrams



In the traditional picture, singularity is part of the future boundary of space-time. Part of the incoming state on $\mathcal{I}_L^-$ falls into it and so the S-matrix from $\mathcal{I}_L^-$ to $\mathcal{I}_L^+$ fails to be unitary. In the ATV scenario the S-matrix is unitary because there is no singularity and $\mathcal{I}_R^+$ is 'as large as' $\mathcal{I}_L^-$; no modes have be traced over.